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CONVECTIVE HEAT-TRANSFER COEFFICIENTS FROM A SOLUTION OF THE CONDUCTION EQUATION FOR A WALL SEPARATING TWO FLUIDS, ONE HAVING AN OSCILLATING TEMPERATURE

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16. Abstract

The temperature response of a wall which separates two fluids, one having a sinusoidally oscillating temperature, has been analytically determined. The solutions show that the convective heat-transfer coefficients on both sides of the wall can be determined experimentally if a single wall temperature along with the hot fluid and coolant temperatures are known as functions of time. From traces of these temperatures, the frequency of oscillation and either the amplitude ratio or the phase lag angle between the forced fluid and wall temperatures are measured. These are used in the solutions for the coefficients. The method is particularly applicable to rocket engines.

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SUMMARY

The temperature response of a wall which separates two fluids, one having a sinusoidally oscillating temperature has been analytically determined. Two mathematical solutions are presented: one uses thermal properties distributed through the wall; the other assumes that the wall thermal properties can be lumped.

The solutions show that the convective heat-transfer coefficients on both sides of the wall can be determined experimentally if a single wall, the hot fluid, and coolant temperatures are known as a function of time. From traces of these temperatures the frequency of oscillation and either amplitude ratio or the phase lag angle between the forced fluid and wall temperatures are measured. These values are used in the solutions for the coefficients. The method is particularly applicable to rocket engines and is generally applicable to heat exchangers.

Charts are included to allow graphical solutions for the distributed-wall-property case. Comparisons of the two mathematical solutions are made which indicate that it may be possible to use the simpler lumped-wall-property solution. This requires, among other things, thin walls and low forcing frequencies. The lumped-wall-property solution may be applied with greater accuracy when the wall temperature is measured at the midpoint of the wall.

INTRODUCTION

Transient and steady-state analyses have been used to design calorimeters for determining convective heat transfer in heat exchangers, rocket engines, and aerodynamic heat-transfer studies. The steady-state calorimeter of reference 1 makes use of the temperature gradient in a material of known conductivity and geometry; the transient type (ref. 2) uses the temperature response of a material to a change in driving temperature.

The temperature response of a wall having one side insulated and the other exposed to a fluid with a sinusoidally varying temperature has been investigated by Anderson (ref. 3) and this author (ref. 4). Equations for the convective heat-transfer coefficients were derived in terms of the wall material properties, frequency, and phase lag angle between the force fluid temperature and the responding wall temperature. The application of these equations is limited to walls having either one side insulated or having fluid temperatures and convective heat-transfer coefficients that are equal on both sides of the wall.

Of more general interest is the problem of determining the heat-transfer coefficients in a heat exchanger or cooled rocket engine. This problem can also be solved, by sinusoidally varying the temperature of the hot fluid or the coolant and measuring the temperature response of the wall. The combustion temperature in a rocket engine, for example, can be oscillated by changing the ratio of the oxidizer-to-fuel mass flow rates while maintaining the total propellant mass-flow rate constant. This will require separate coolant and propellant systems so that the mass flow rates that affect the heat-transfer coefficients can be held constant. This uncoupled system is completely reasonable from a research point of view.

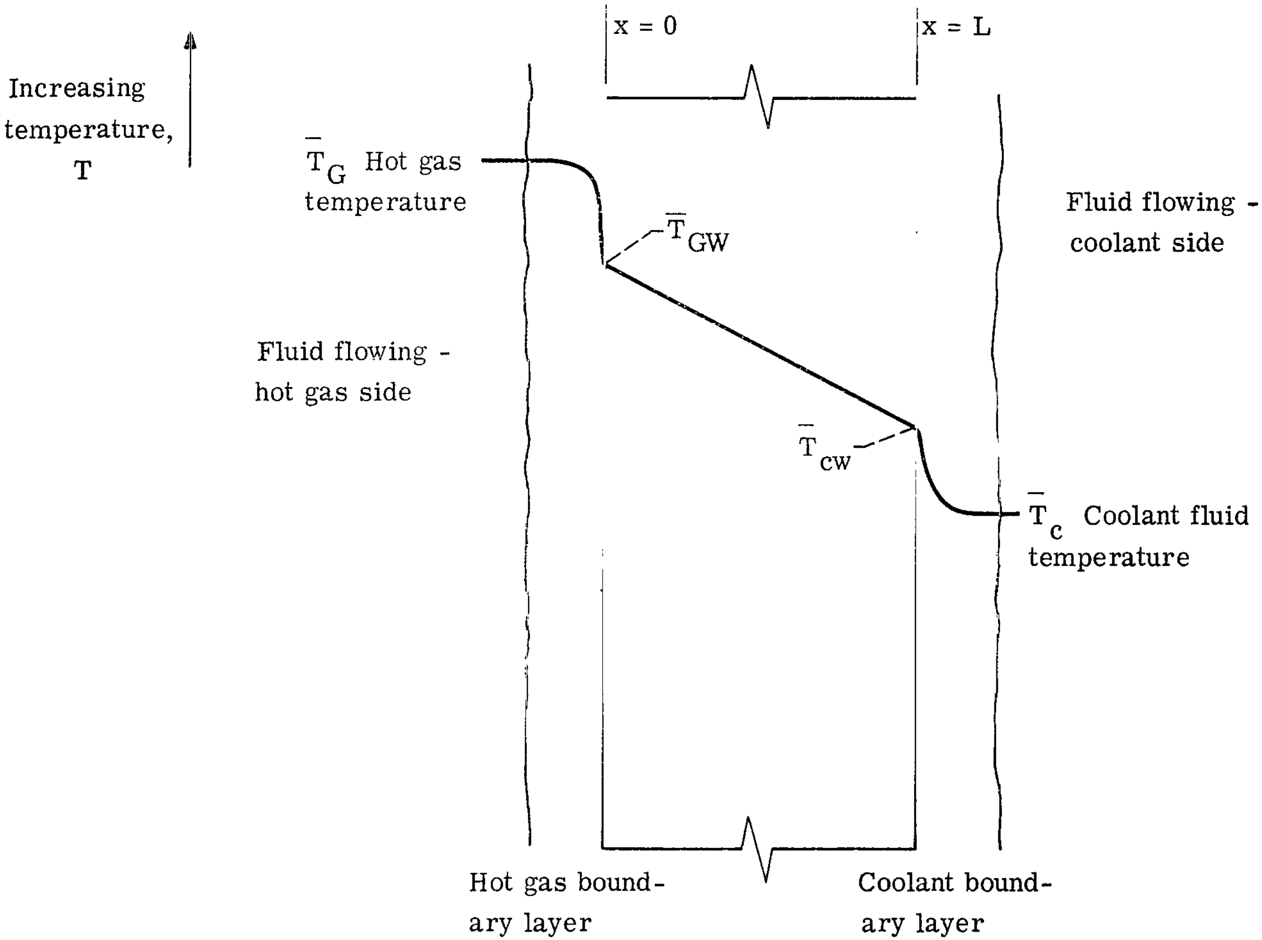
The purpose of this analytical study was to derive the equations for the convective heat-transfer coefficients in terms of the sinusoidally varying temperature response of a wall which separates two fluids, one having a sinusoidally oscillating temperature. This study was performed at NASA Lewis in conjunction with a rocket nozzle heat-transfer program.

STATEMENT OF THE PROBLEM

Figure 1(a) shows a wall that separates two moving fluids. The hot-gas-side fluid temperature T_G is greater than the coolant temperature T_c, causing heat to flow through the wall in the positive x direction. The problem is to determine the convective heat-transfer coefficients if either the hot-gas or coolant temperature is varied sinusoidally and if the temperature response of the wall is measured at only one point. Figure 1(b) shows what the wall temperature might look like at any given instant in time.

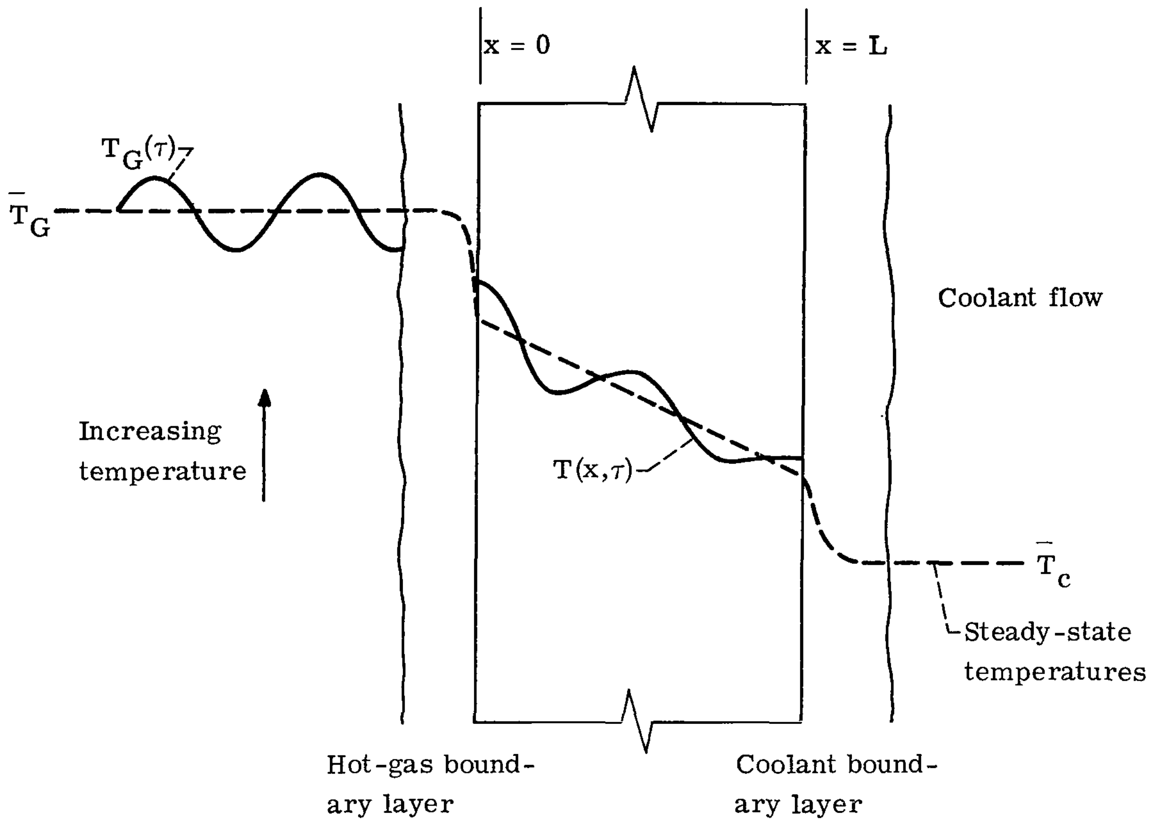
METHOD OF ATTACK

If the hot-gas temperature is made to vary sinusoidally, the wall temperature will



(a) Wall temperature distribution shown at time zero before start of hot-gas temperature oscillation.

Figure 1. - Basic heat-transfer model.



(b) Transient temperature response of plate to sinusoidally forced fluid temperature.

Figure 1. - Continued.

respond sinusoidally but will lag the driven temperature by an angle φ. In addition, the amplitude of the wall temperature oscillation will be less than that of the driven temperature. Both the phase lag φ and the ratio of the amplitude of the wall temperature to the amplitude of the driven temperature $\theta/\Delta T$ are, among other things, functions of the convective heat-transfer coefficients h. Finding the relation between the convective heat-transfer coefficients and the phase lag φ, or the amplitude ratio $\theta_m/\Delta T$, is required in order to solve for the coefficients.

To find this relation, the temperature response of the wall to a sinusoidally driven fluid temperature is derived. From this solution and steady-state heat-transfer conditions (which relate the ratio of the convective heat-transfer coefficients to the ratio of the temperature drops between fluids and wall), the absolute values of the coefficients can be determined as functions of either phase lag or amplitude ratio. The quantities that must be known as a function of time are the hot-gas temperature T_G, the wall temperature at any point x $(T(x, \tau))$ and the coolant temperature T_c. From these quantities either the phase lag or the amplitude ratio can be determined as well as the ratio of the convective heat-transfer coefficients. With the ratio of the coefficients, the frequency of the temperature oscillation, the wall material properties, and either φ or $\theta_m/\Delta T_G$, the convective heat-transfer coefficients h_G and h_c can be calculated.

It is also possible to calculate the coefficients by oscillating the coolant temperature sinusoidally. This approach requires the measurement of the same quantities as when the hot fluid temperature is oscillated.

FORMULATIONS AND SOLUTIONS OF THE PROBLEM

Two analytical solutions have been found, each of which relate the heat-transfer coefficients to either the phase lag or the amplitude ratio. The first solution (case 1) uses the second-order partial differential equation for transient, one-dimensional, heat conduction in a plate. The boundary conditions are that one surface is heated convectively and the other surface is cooled convectively. The second solution (case 2) uses a lumped wall property approach. The differential equation used in case 2 is obtained by equating the heat stored in the wall as a function of time to the difference between the heat transferred to one side of the plate convectively and the heat taken from the other side of plate convectively. The case 2 solution for the wall temperature can be broken into three parts: the starting transient, an offset of the average value from the value before the start of the oscillation, and the steady-state oscillation. The steady-state oscillating part of the case 2 solution is used extensively in this work.

Solutions for both case 1 and case 2 are given in this report so that the reader can determine for himself when the simpler, though less accurate, case 2 solution is applicable.

Distribution Wall Properties (case 1)

The following assumptions have been used to obtain the case 1 solution:

(1) The heat flows through the plate in the x direction only (one-dimensional heat conduction).

(2) The wall or plate properties (density, specific heat, and thermal conductivity) are constant.

(3) The convective heat-transfer coefficients on both sides of the wall are constant.

(4) The coolant temperature is constant.

Determining the validity of assumption (1) is left to the experimenter with his particular application since no general comments are possible as to when one-dimensional heat flow is present. The second assumption, that of constant wall properties, will depend on the wall material and the temperature through the wall including the amplitude of the oscillations. The wall temperature amplitude can be minimized so that the assumption of constant wall properties as a function of time should be applicable. If the wall is thin enough or the heat flux low enough, the wall properties may not change appreciably from one surface to the other. The third assumption, constant coefficients, appears justified because the coefficients are usually not strong functions of pressure or temperature. They are, however, strong functions of the mass velocity and for that reason the mass flow rate of the fluids are held constant. In the experimental rocket engine this is accomplished by uncoupling the coolant flow rate from the propellant flow rate by use of separate systems. This allows a constant coolant flow rate to be set. The total mass flow rate of the propellants (oxidizer plus fuel) is held constant but in order to vary the combustion temperature the ratio of the individual propellant mass flow rate is varied. The validity of assumption (4) can be insured by limiting the amplitude of the hot fluid temperature oscillation to a value that forces the amplitude of the wall temperature oscillation to be just adequately measureable. This measurement can be aided by the use of amplifiers that would allow the use of smaller hot-fluid amplitudes. With small wall-temperature amplitudes the coolant temperature will remain practically constant.

The controlling differential equations and boundary conditions are given next. The one-dimensional transient heat conduction equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \tag{1}$$

is solved for the temperature distribution in a wall which has convective heat transfer over its two surfaces (fig. 1). (Symbols are defined in appendix A.) The hot-gas side fluid temperature is driven sinusoidally and is given in equation form by

$$T_G = \Delta T_G e^{-i\omega\tau} + \overline{T}_G \tag{2}$$

where $\overline{T}_G$ is given, in the rocket engine application by the usual chemical calculational techniques and the time dependent term is sensed by the chamber pressure. It is not necessary to know the numerical value of ΔT_G if the phase lag is used for calculating the coefficients since the phase determination is independent of the absolute value of the temperature oscillation. If the amplitude ratio is used for calculating the coefficients the ΔT_G must be computed from the change in chamber pressure. The boundary condition at the hot-gas side of the wall surface $x/L = 0$ equates the heat transferred by convection to that conducted away from the surface into the wall. This is given in equation form as

$$h_G \left[T_G - T(0, \tau)\right] = -K \frac{\partial T(0, \tau)}{\partial x} \tag{3}$$

The boundary condition at the coolant surface ($x/L = 1$) equates the heat conducted to the surface to the heat transferred convectively to the coolant. In equation form this is given by

$$h_c \left[T(L, \tau) - T_c\right] = - \frac{K \partial T(L, \tau)}{\partial x} \tag{4}$$

The solution for the wall temperature as a function of time and location within the wall consists of the sum of the transient $\theta(x, \tau)$ and steady-state $T_s(x)$ solutions. The steady-state solution is found by setting $\partial T/\partial \tau$ equal to zero in equation (1) and assuming the wall temperature at $x/L = 0$ and 1 to be given as $\overline{T}_{GW}$ and $\overline{T}_{cw}$, respectively. This yields the usual linear temperature distribution with distance and can be written as

$$\overline{T}_s(x) = (\overline{T}_{cw} - \overline{T}_{GW}) \frac{x}{L} + \overline{T}_{GW} \tag{5}$$

The transient solution $\theta(x, \tau)$ is found by assuming the usual product solution: one a function of time only $\overline{F}(\tau)$, and the other a function of the location within the wall measured from the heated side of the wall $X(x)$. Equations (1), (3), and (4) are modified to account for the change in variable from $T(x, \tau)$ to $\theta(x, \tau)$. The details of the solution of equation (1) using its pertinent boundary conditions (eqs. (2) to (4)) are shown in appendix B.

The transient part of the solution $\theta(x, \tau)$ is given in appendix B in complex form by equation (B25) presented here in functional form as

$$\theta(x, \tau) = \frac{\theta_m}{\Delta T_G} P\,(x,\ L,\ \eta,\ \psi_G,\ \psi_c) \exp^{-i(\omega\tau - \varphi)}$$

where

$$\varphi = V\left(\frac{x}{L},\ \eta L,\ \psi_G,\ \psi_c\right)$$

From this equation, the phase lag φ (eq. (B26)) and the amplitude of the wall temperature oscillation (eq. (B27)) are derived. Equations (B28) to (B33) define the parameters used in equations (B25) to (B27). Equations (B25) to (B27) are rewritten in nondimensional form as equations (B34), (B36), and (B37), respectively. The governing nondimensional parameters which appear in the transient solution equation (B34) are ψ_G and ψ_c, the convective heat-transfer parameters; ηL, the wall properties-thickness-frequency parameter; and x/L, the location at which the wall temperature is being considered. The equations for phase lag (eq. (B36)) and amplitude ratio (eq. (B37)) are given as functions of the nondimensional parameters. Either one of these equations gives a relation between the desired coefficients but, since these equations are derived from equation (B34), it is impossible to solve them simultaneously for the desired coefficients. Instead another independent relation must be found.

The required relation is derived from the steady-state (or mean) condition that requires the heat transferred convectively to the wall to equal the heat transferred convectively to the coolant. This relation is expressed mathematically as

$$\frac{h_c}{h_G} = \frac{\overline{T}_G - \overline{T}(0)}{\overline{T}(L) - \overline{T}_c} \tag{6}$$

From the definition of ψ

$$h = \frac{K\eta}{\psi} \tag{7}$$

The ratio of convective heat-transfer coefficients in equation (6) is defined as R. Calculating the ratio R using equation (7) gives

$$R = \frac{\psi_G}{\psi_c} \tag{8}$$

where R is also given by

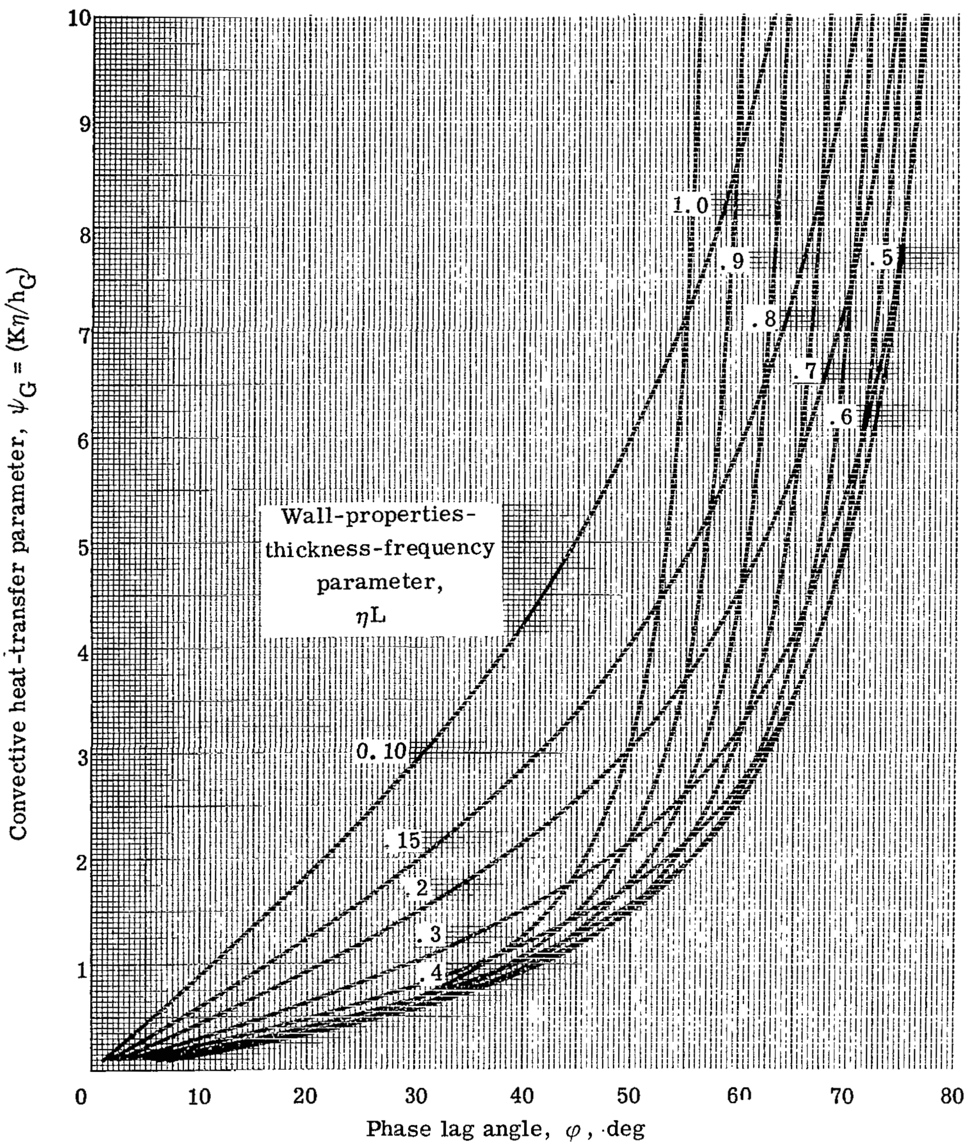
$$R = \frac{\overline{T}_G - \overline{T}(0)}{\overline{T}(L) - \overline{T}_c} \tag{9}$$

(Eqs. (8) and (9) are also eqs. (B50) and (B49), respectively.) Since $\overline{T}_G$, $\overline{T}_c$, and a wall temperature have been given which for the present is assumed to be constant allowing $\overline{T}(0) = \overline{T}(L)$, R may be calculated from equation (9). With the value of R known, equation (8) then gives the second required relation between ψ_G and ψ_c. Only one unknown exists in each of equations (B34), (B36), and (B37). Thus, ψ_G and ψ_c can now be calculated either by knowing the phase lag φ and using equation (B36), or by the amplitude ratio and using equation (B37).

The convective heat-transfer parameters ψ_G, calculated as a function of phase lag φ, wall-properties-thickness-frequency parameter ηL, wall temperature location x/L, and heat-transfer coefficient ratio R, using equation (B36), is plotted in figure 2. A second plot of the convective heat-transfer parameters ψ_G calculated as a function of wall temperature amplitude ratio $\theta_m/\Delta T_G$, wall-properties-thickness-frequency parameter ηL, wall temperature location x/L, and heat-transfer coefficient ratio R, using equation (B37), is shown in figure 3. The $R = 0$ charts are included because they show the special case of an insulated surface at $x/L = 1.0$, that is, at $h_c = 0$ (ref. 4). In figure 2 the charts for $x/L = 1.0$ have curves of ψ against φ which appear to give negative values for ψ. This is explained by letting h_G approach infinity so that ψ_G also approaches infinity. As this happens, R approaches zero, which forces φ to approach zero proving that positive values of φ can exist for $\psi = 0$. Figure 3 shows that for $x/L = 1.0$ the curves for ψ are double valued. The choice of values can be made using the phase lag angle solution of figure 2 to pick the correct value.

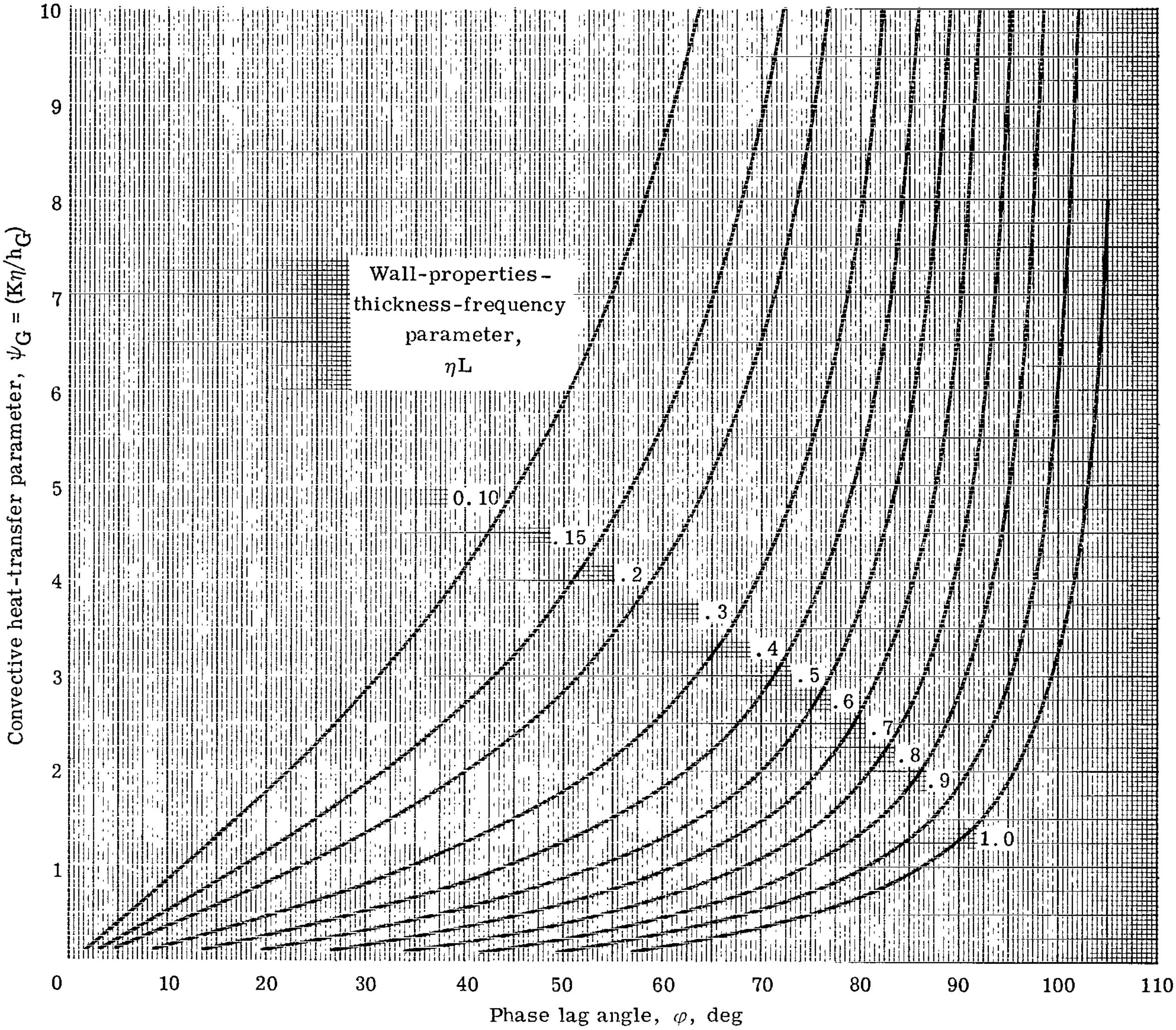
Figures 2 and 3 are included in this report to aid the experimentalist in (1) determining the frequency and amplitude of the hot-gas temperature oscillation that is required to give a measurable wall temperature response, and (2) calculating the convective heat-transfer coefficient. When using the charts to determine wall temperature response, a calculation should be made to determine whether the coolant temperature will oscillate significantly, which would violate the fourth assumption used in the derivation of the solution. An example is given here to show how the heat-transfer coefficients may be calculated using figure 2.

(1) Assume for the moment that the temperature drop across the wall is small. Then the measured wall temperature can be taken equal to $\overline{T}(0)$ and $\overline{T}(L)$. The mean value of the hot fluid $\overline{T}_G$, wall $\overline{T}(0)$, and coolant $\overline{T}_c$ temperatures are substituted in equation (9) and the heat-transfer coefficient ratio R is calculated. The value of R and the location of the measured wall temperature, $x/L = 0$ or 1.0 for these charts, determines which chart in figure 2 is used.



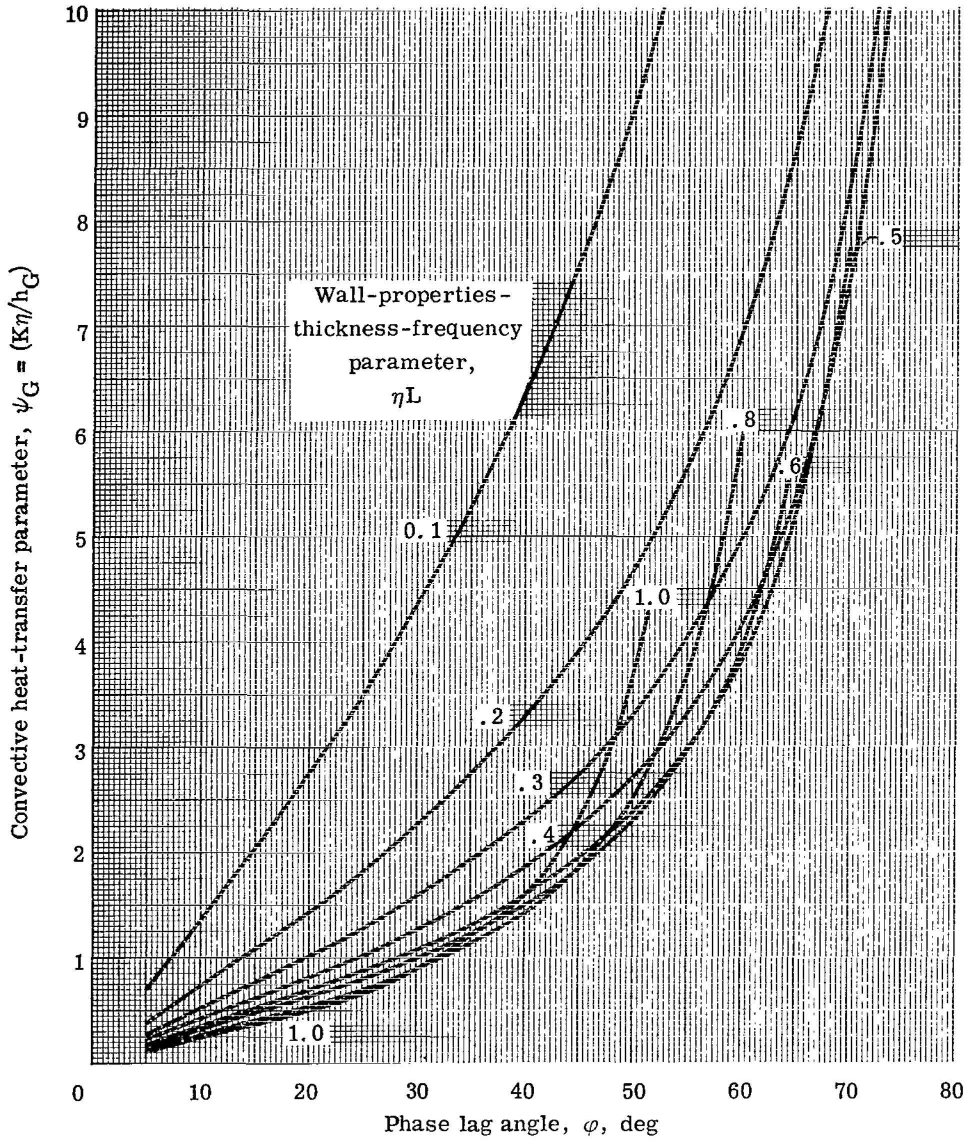
(a) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 0 (insulated wall).

Figure 2. - Convective heat-transfer parameter as function of phase lag angle.



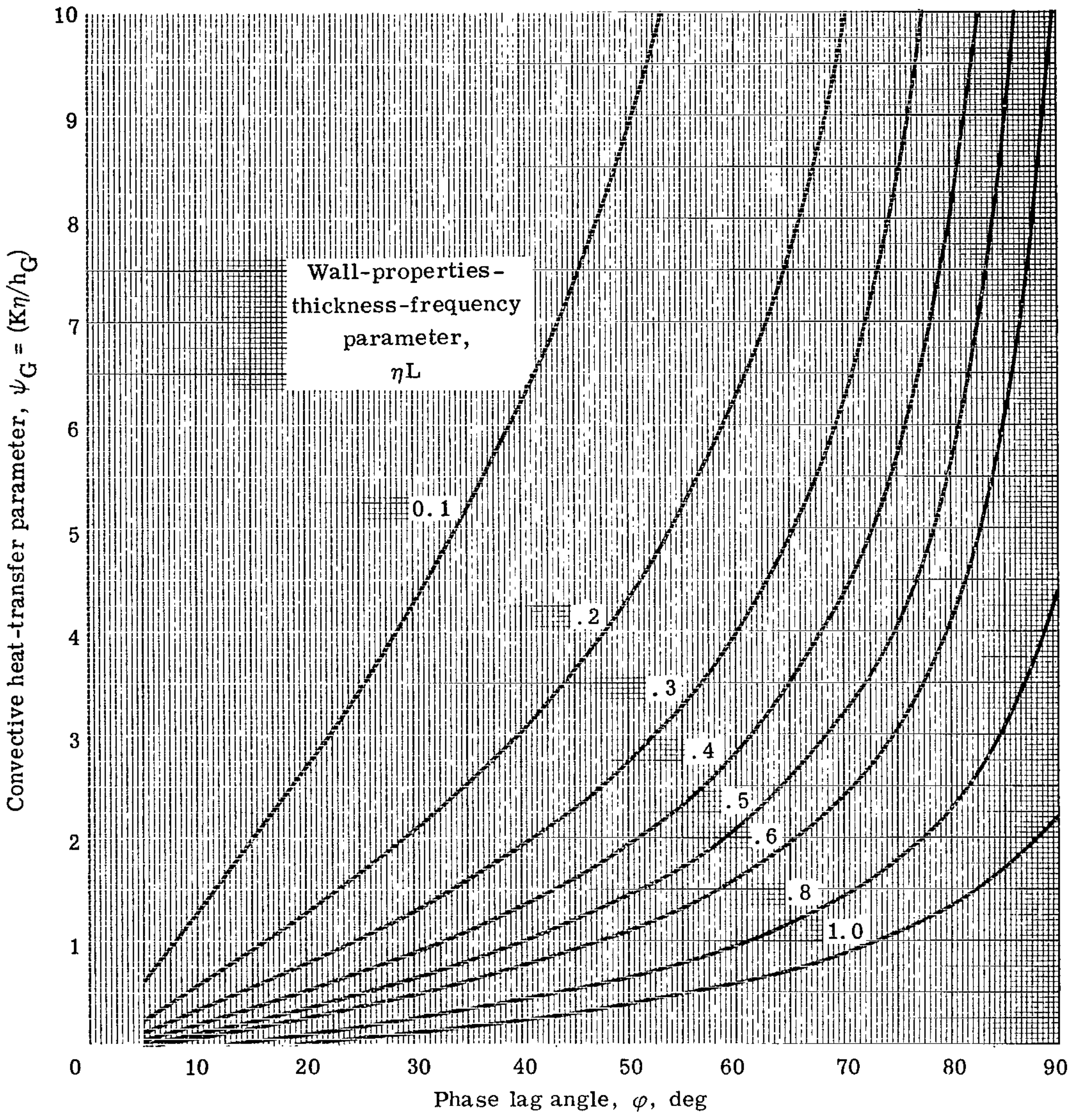
(b) Wall temperature measurement location, x/L = 1.0; ratio of convective heat-transfer coefficients, 0 (insulated wall).

Figure 2. - Continued.



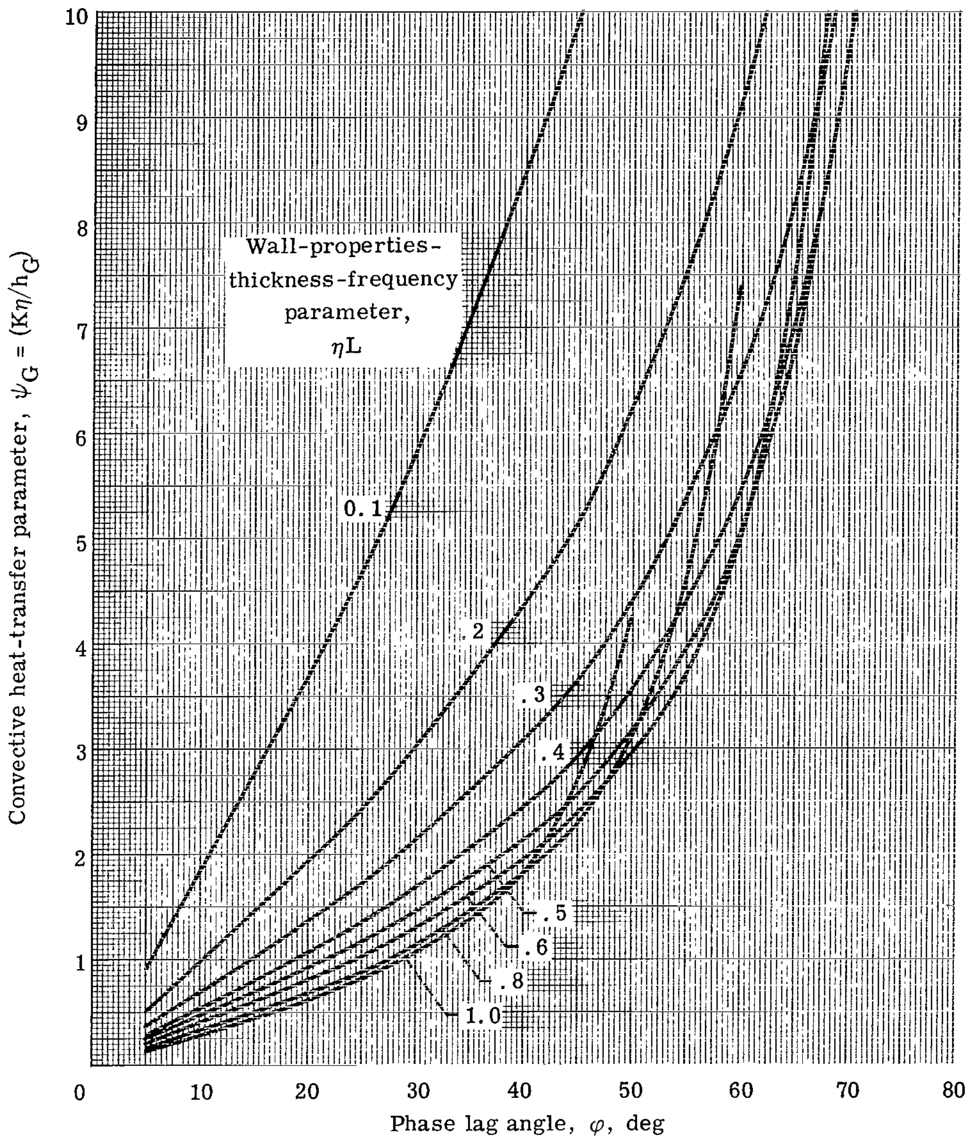
(c) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 0.5.

Figure 2. - Continued.



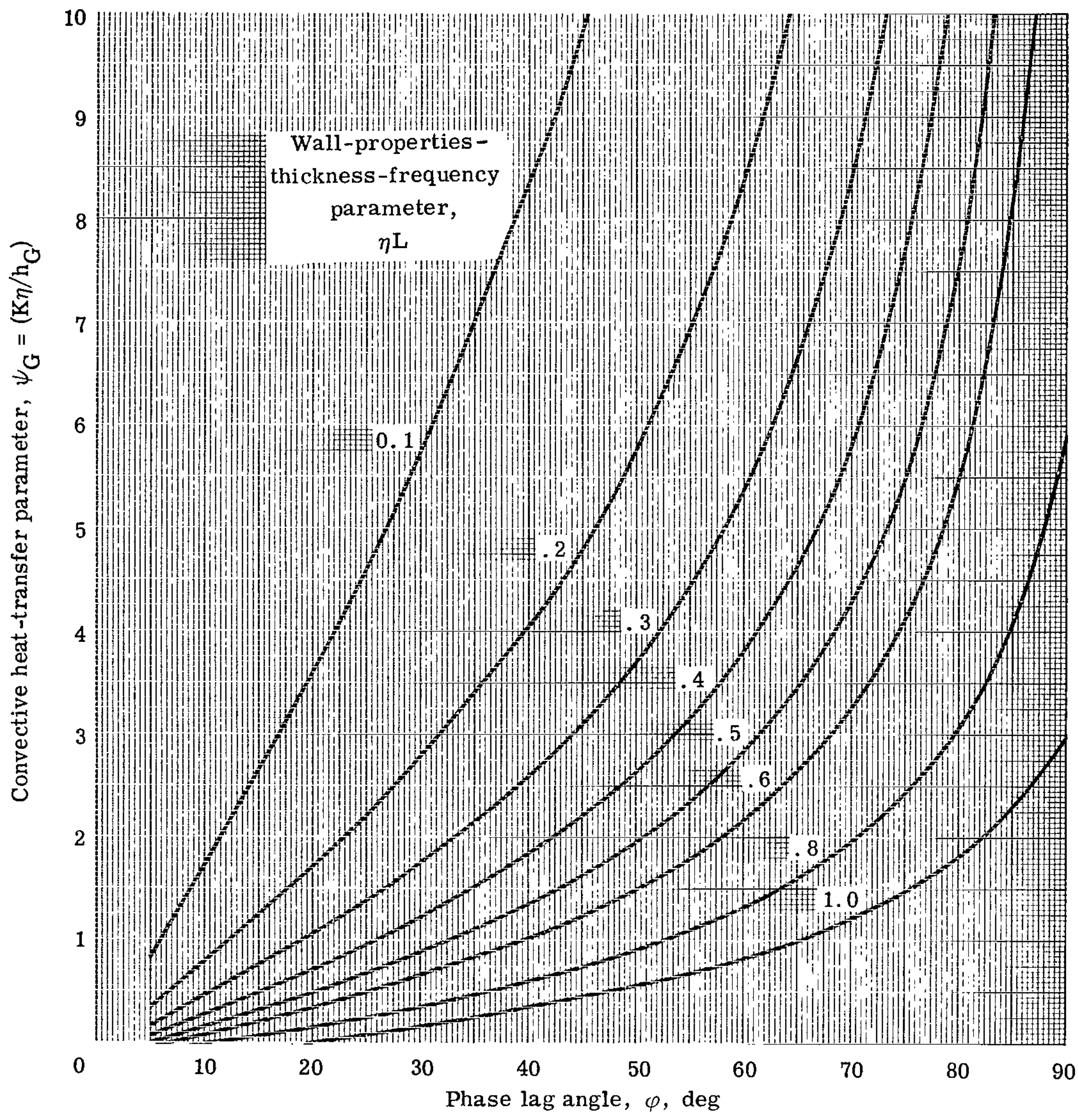
(d) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 0.5.

Figure 2. - Continued.



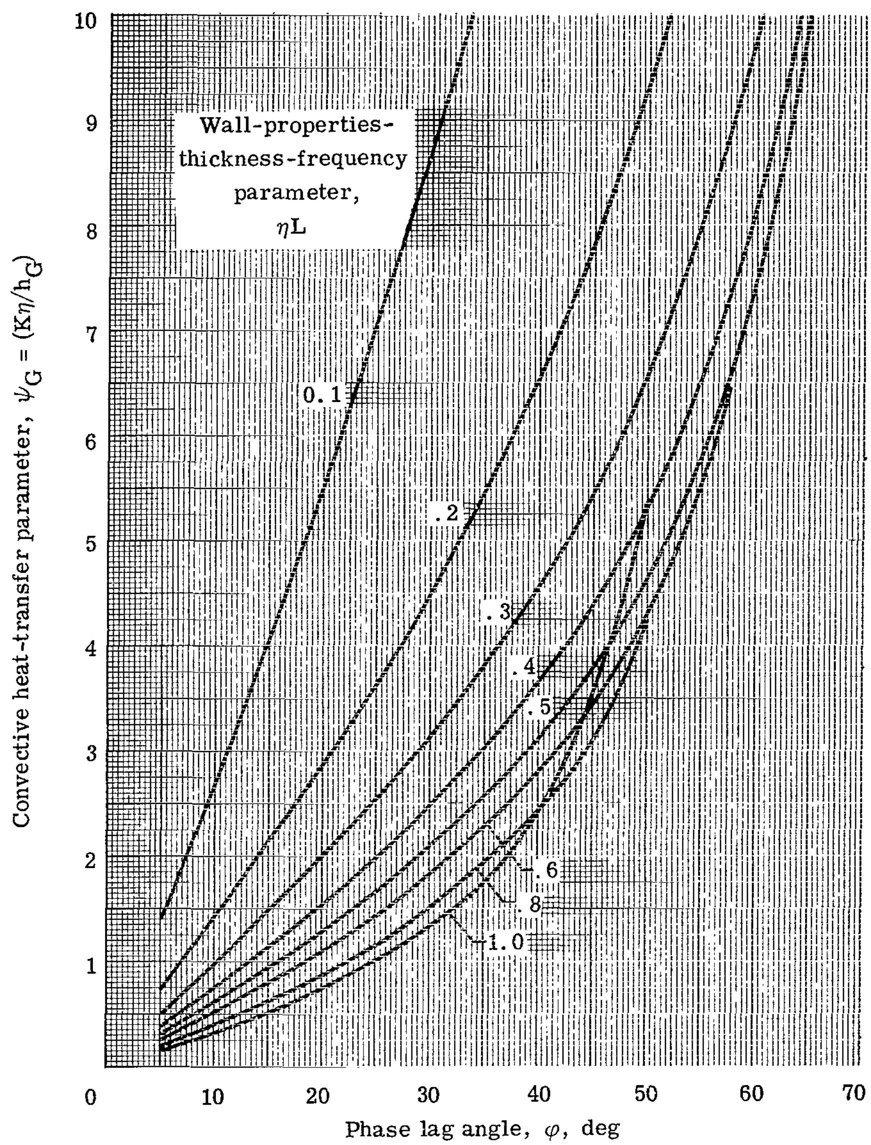
(e) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 1.0

Figure 2. - Continued.



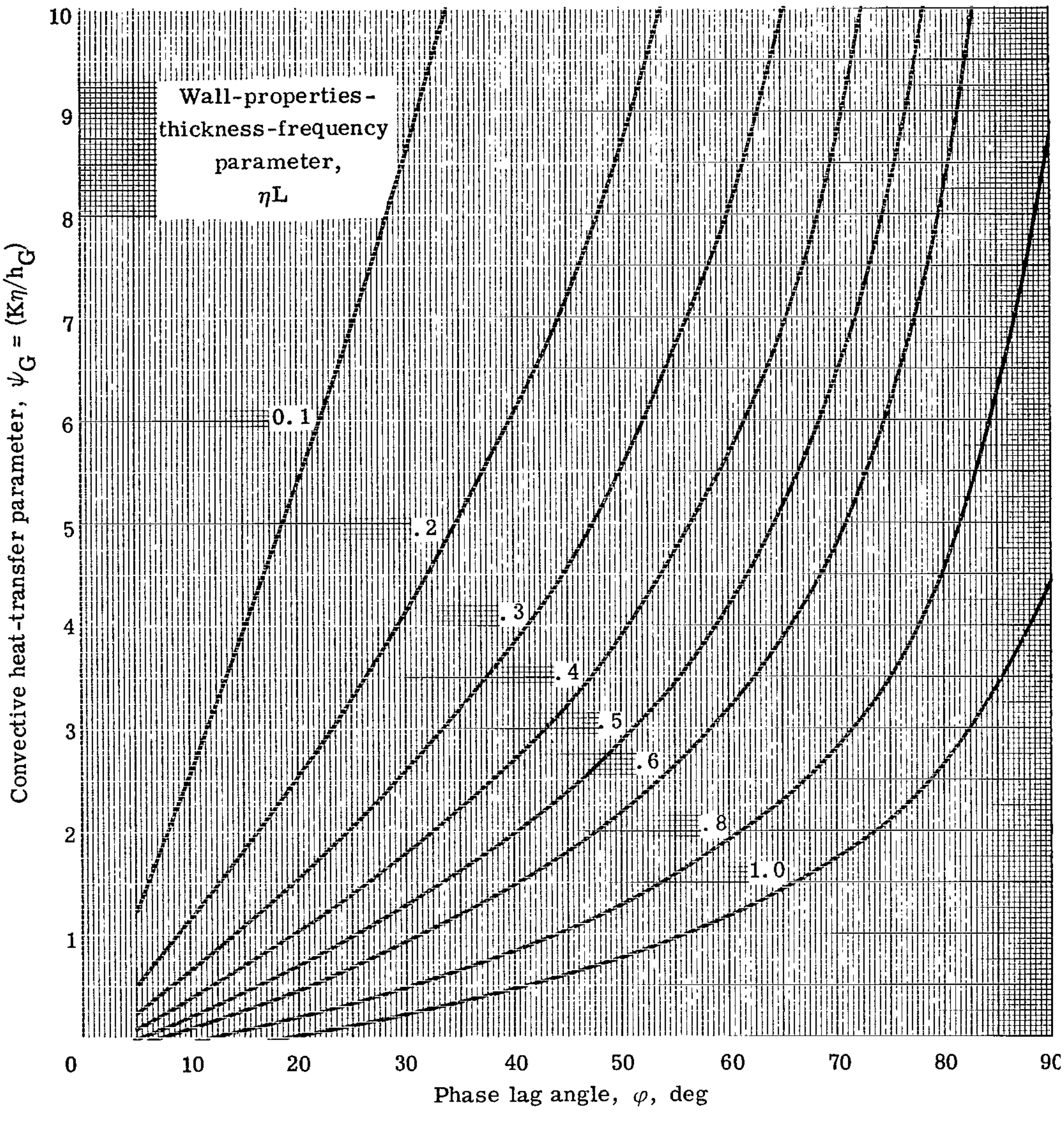
(f) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 1.0.

Figure 2. - Continued.



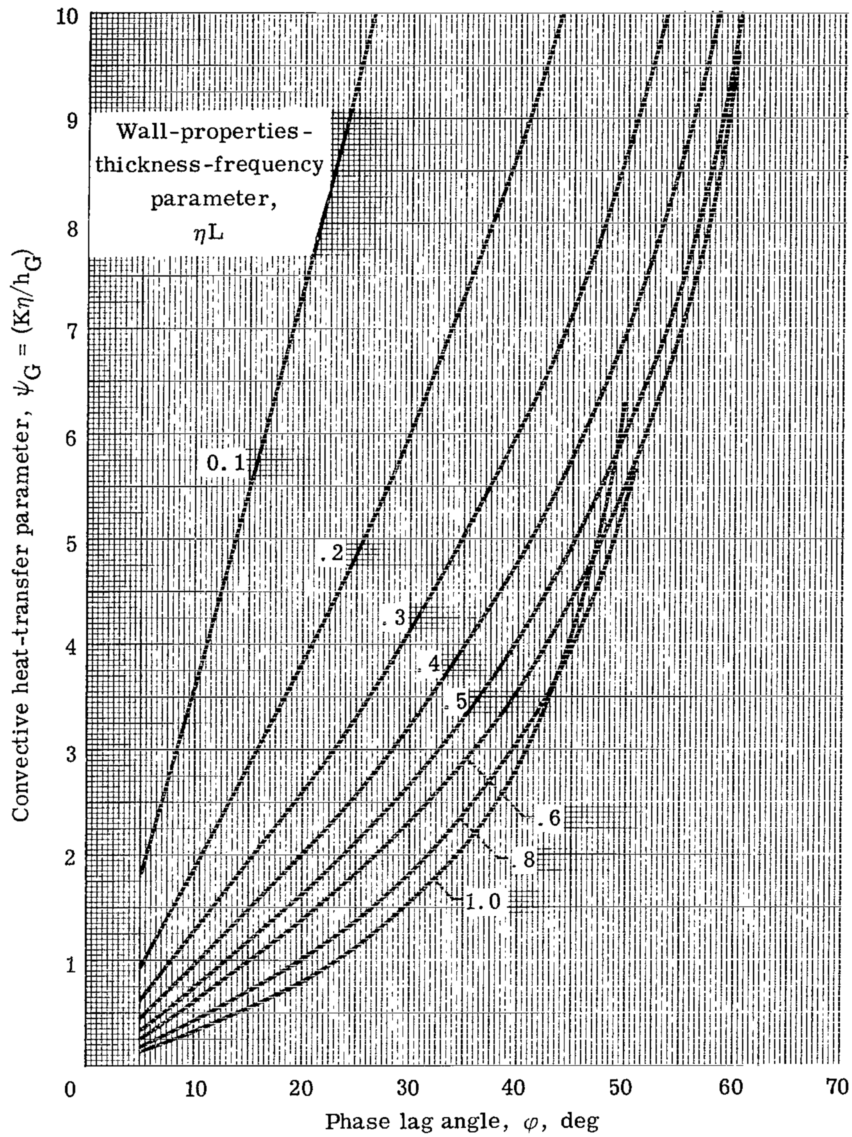
(g) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 2.0.

Figure 2. - Continued.



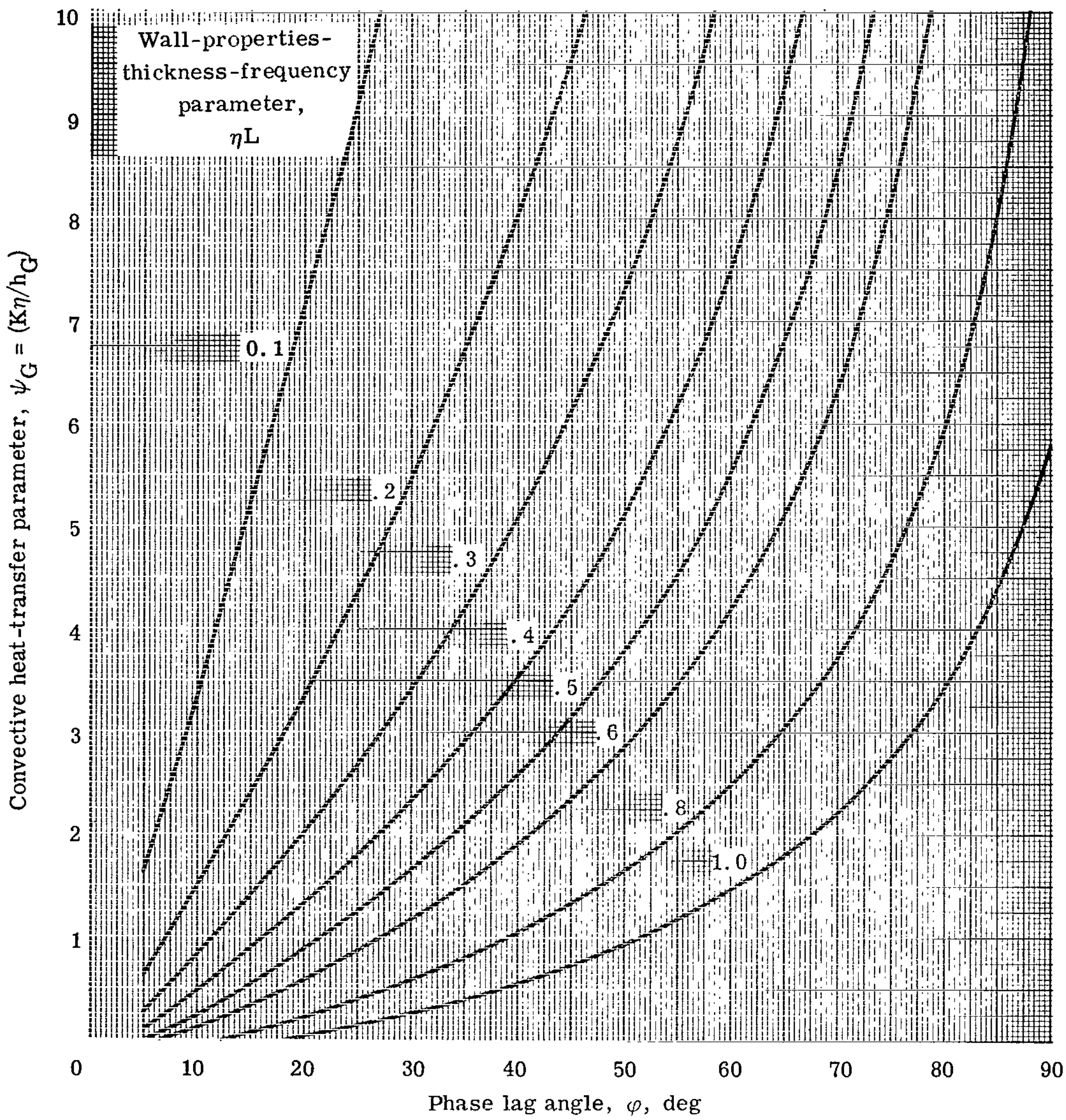
(h) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 2.0.

Figure 2. - Continued.



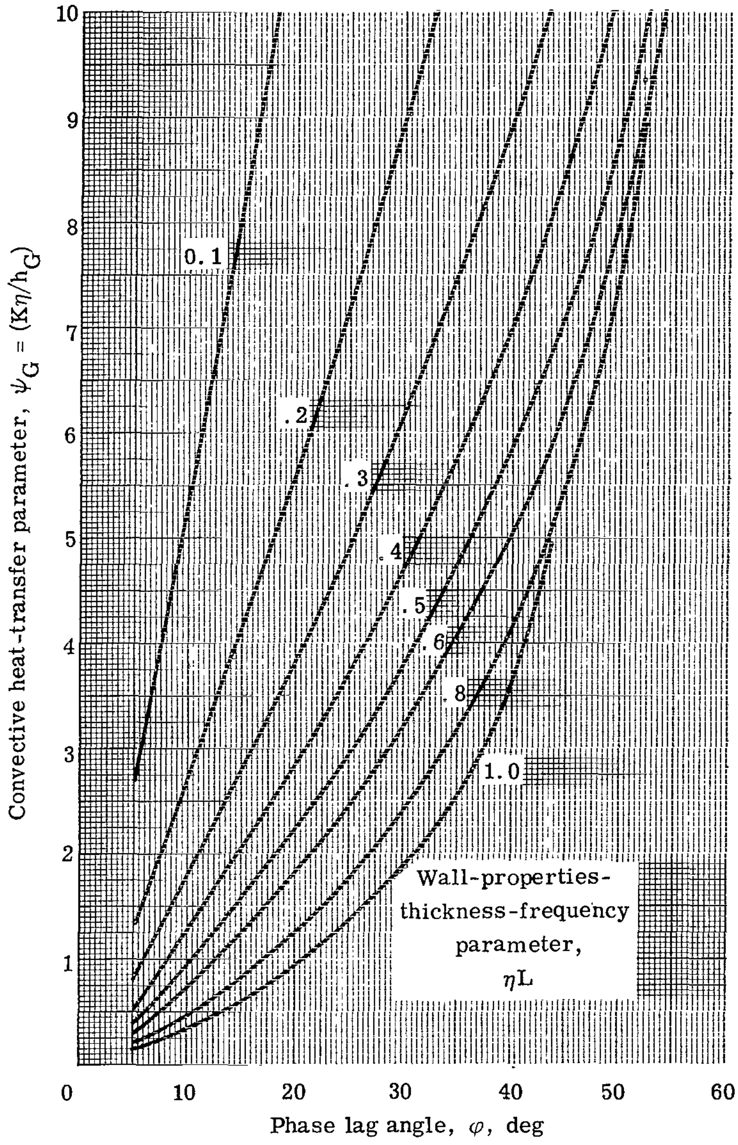
(i) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 3.0.

Figure 2. - Continued.



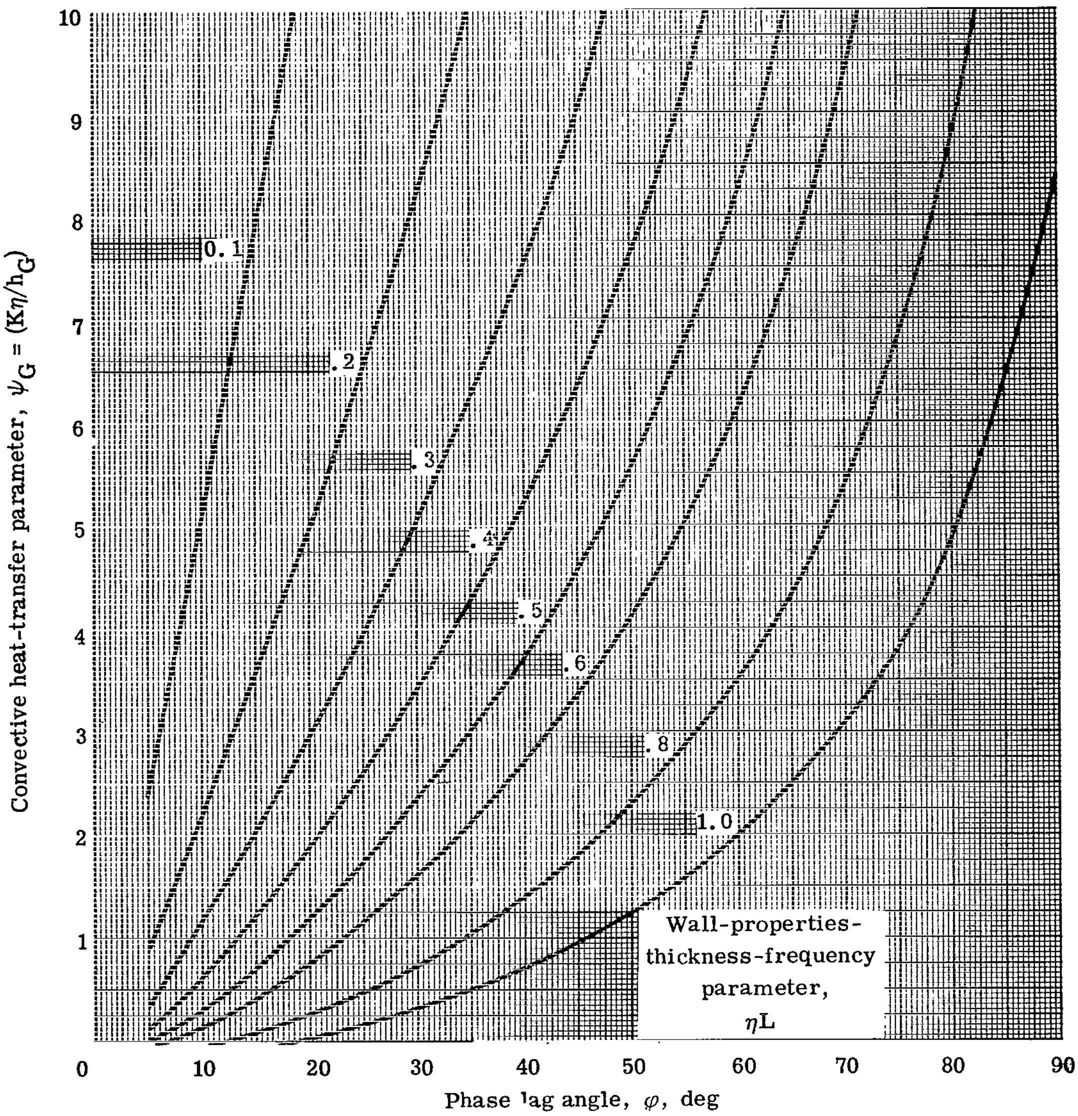
(j) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 3.0.

Figure 2. - Continued.



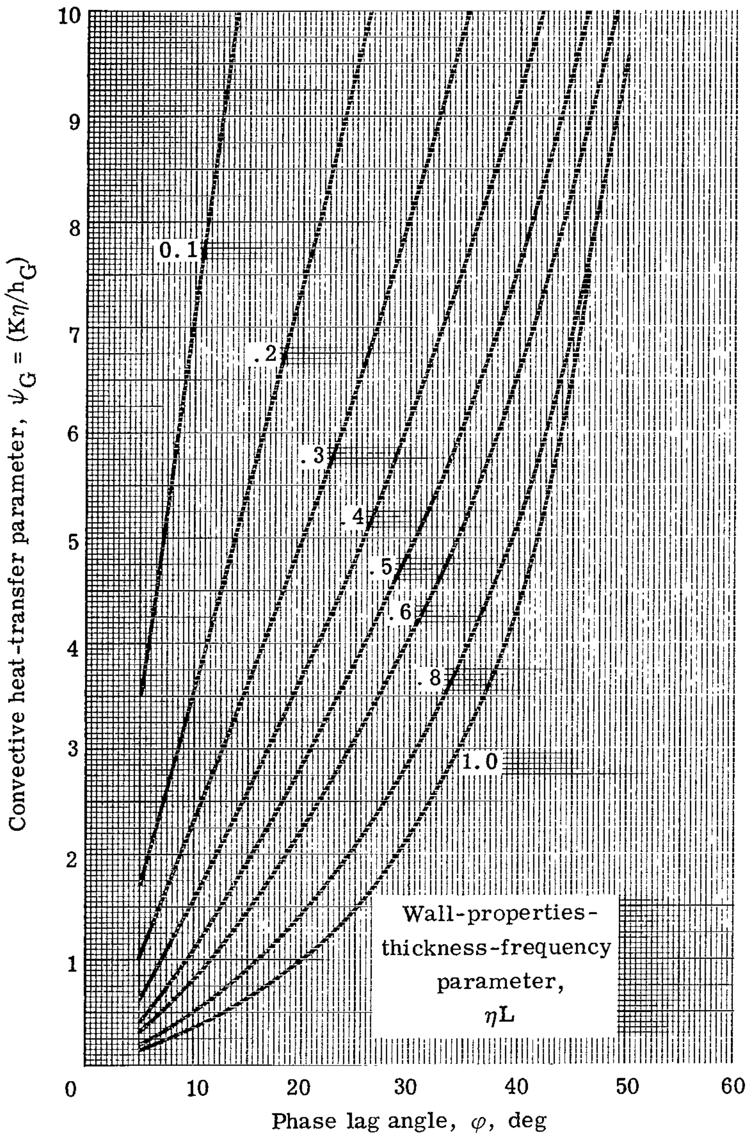
(k) Wall temperature measurement location, x/L = 0; ratio of convective heat-transfer coefficients, 5.0.

Figure 2. - Continued.



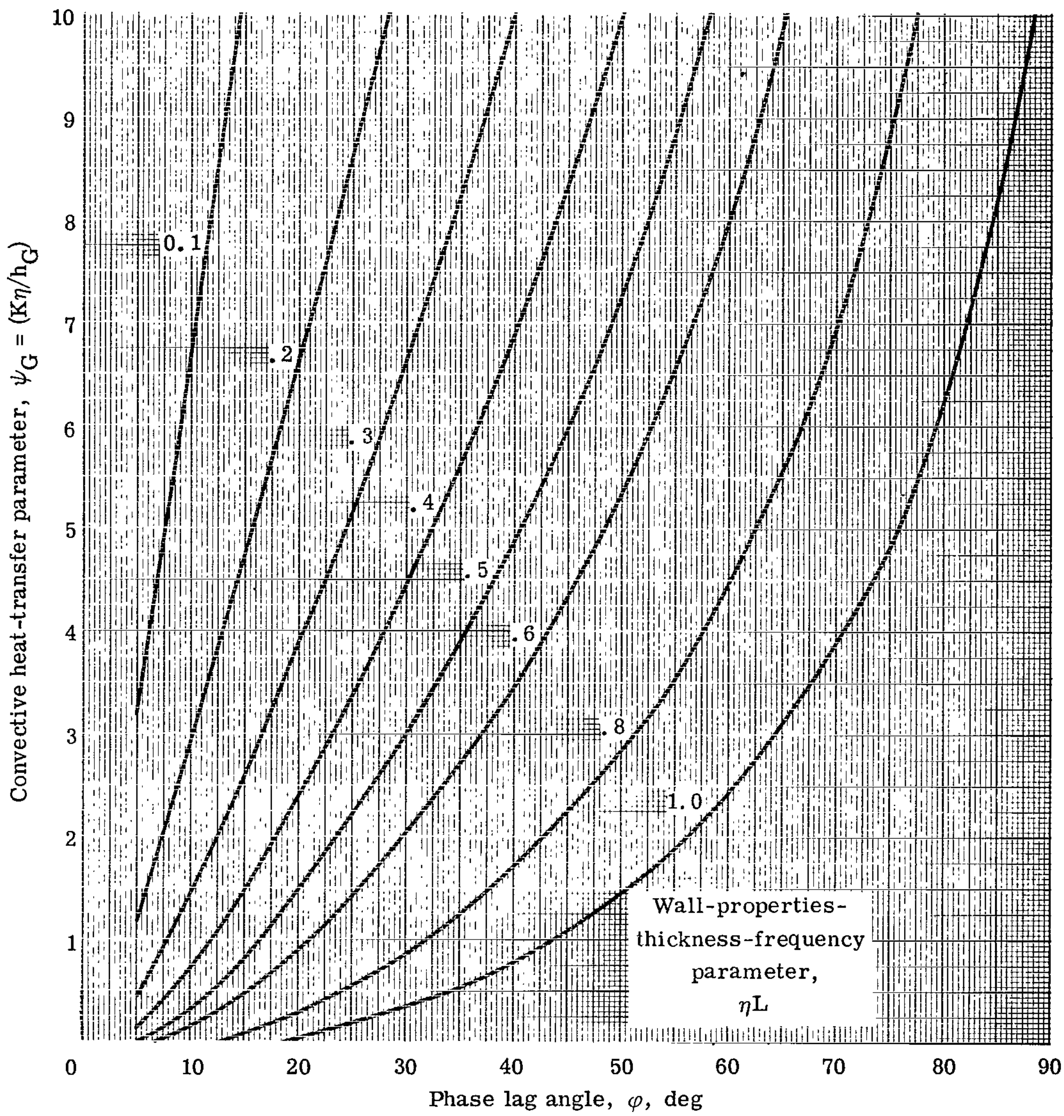
(l) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 5.0.

Figure 2. - Continued.



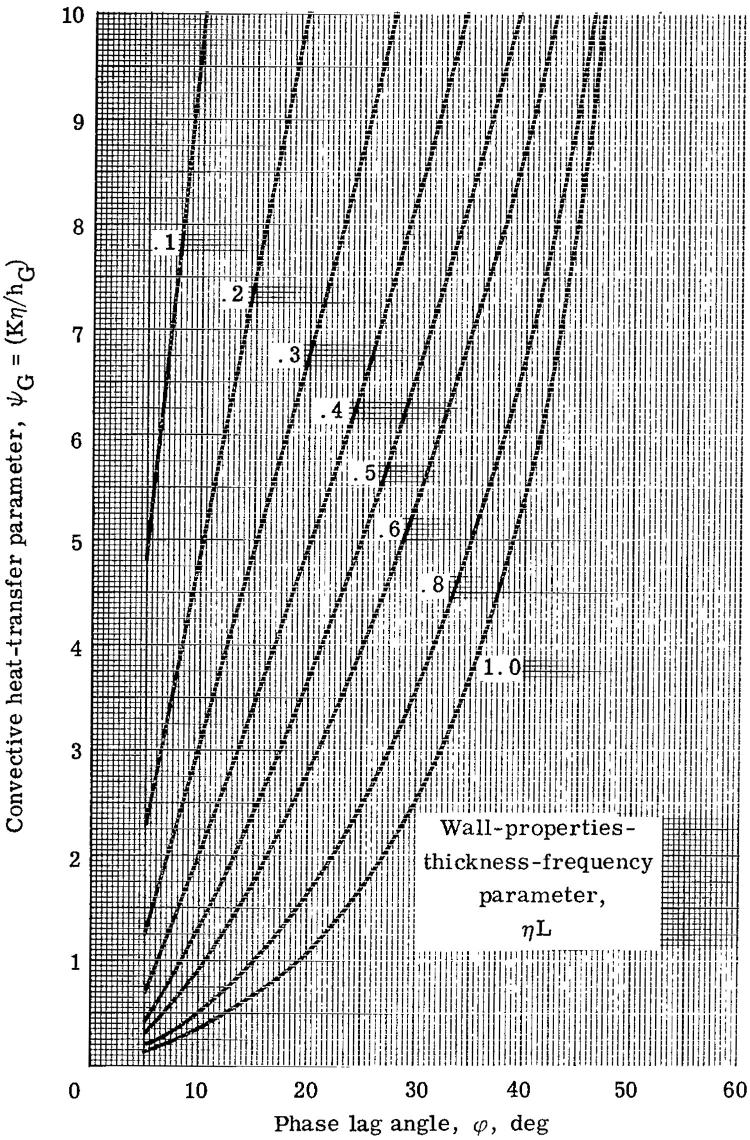
(m) Wall temperature measurement location, x/L = 0; ratio of convective heat-transfer coefficients, 7.0.

Figure 2. - Continued.



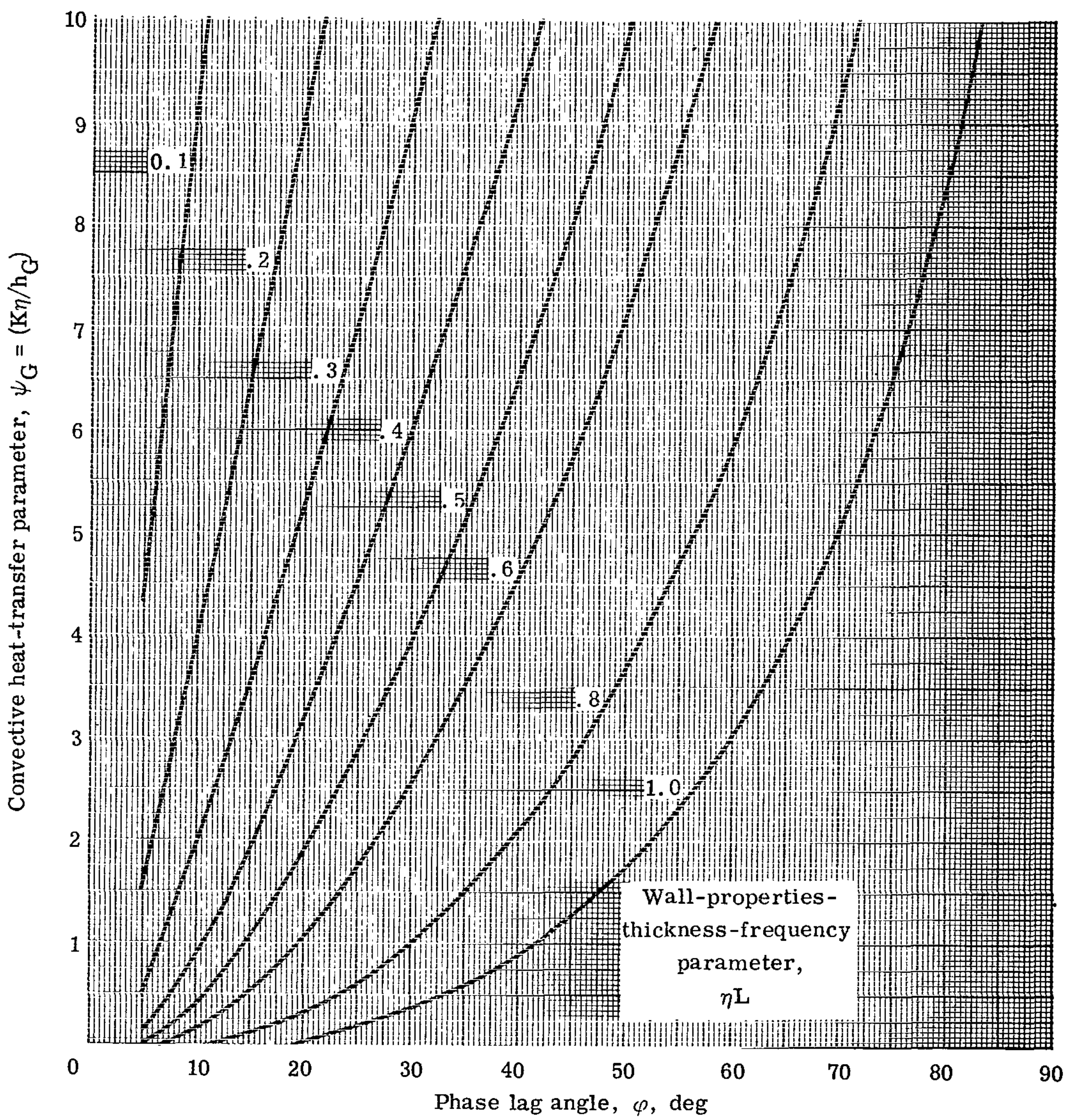
(n) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 7.0.

Figure 2. - Continued.



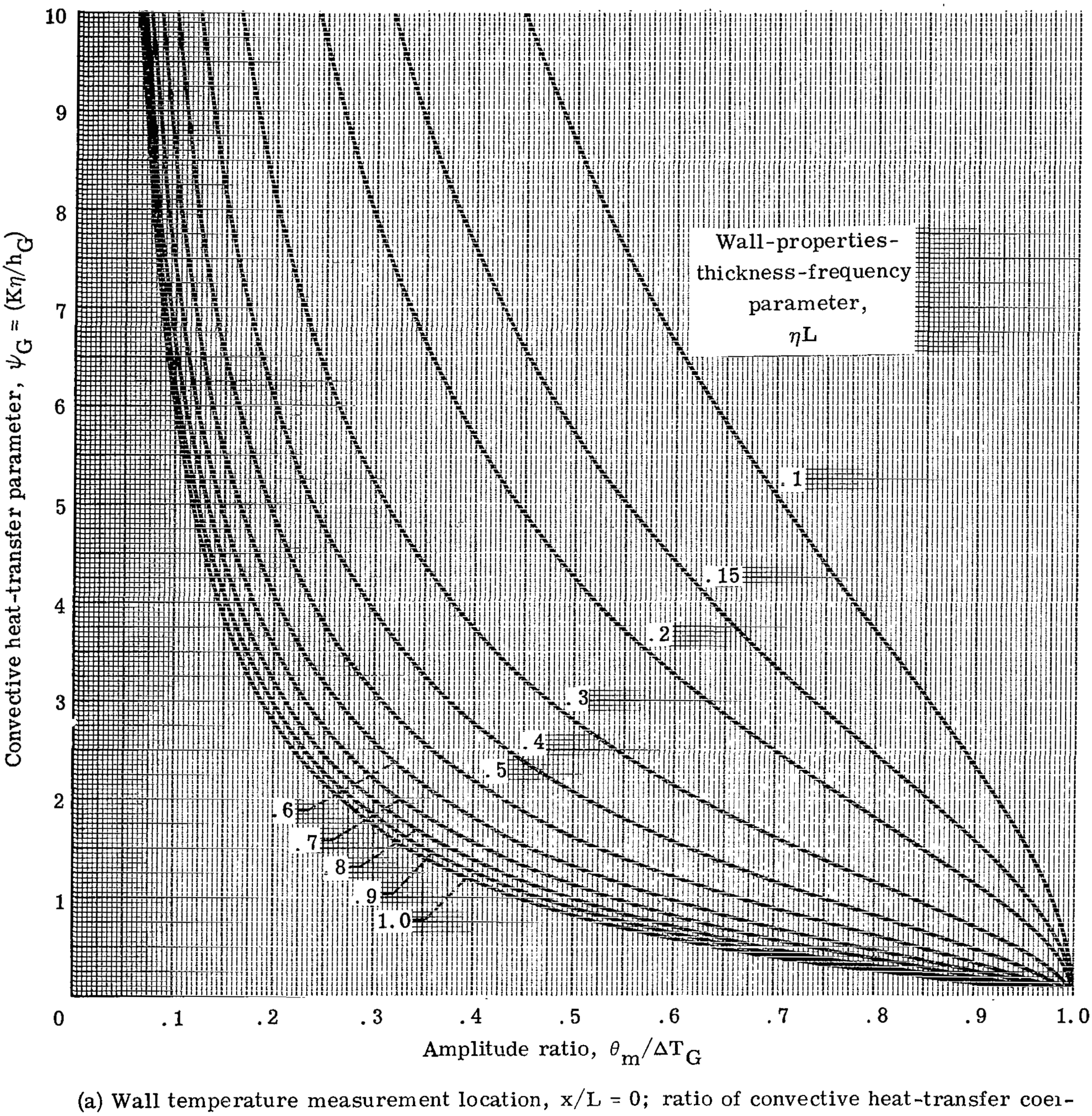
(o) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 10.0.

Figure 2. - Continued.



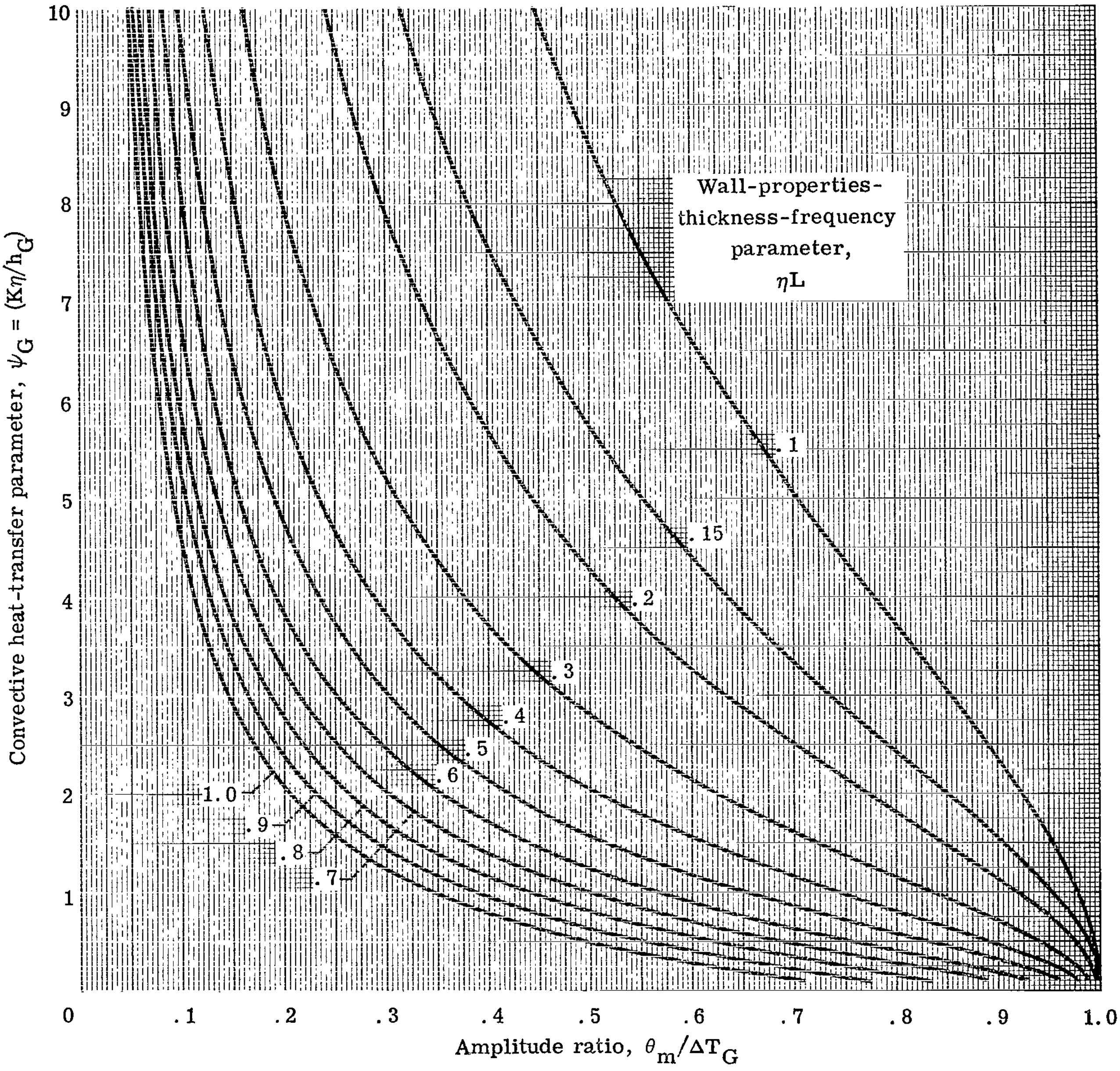
(p) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 10.0.

Figure 2. - Concluded.



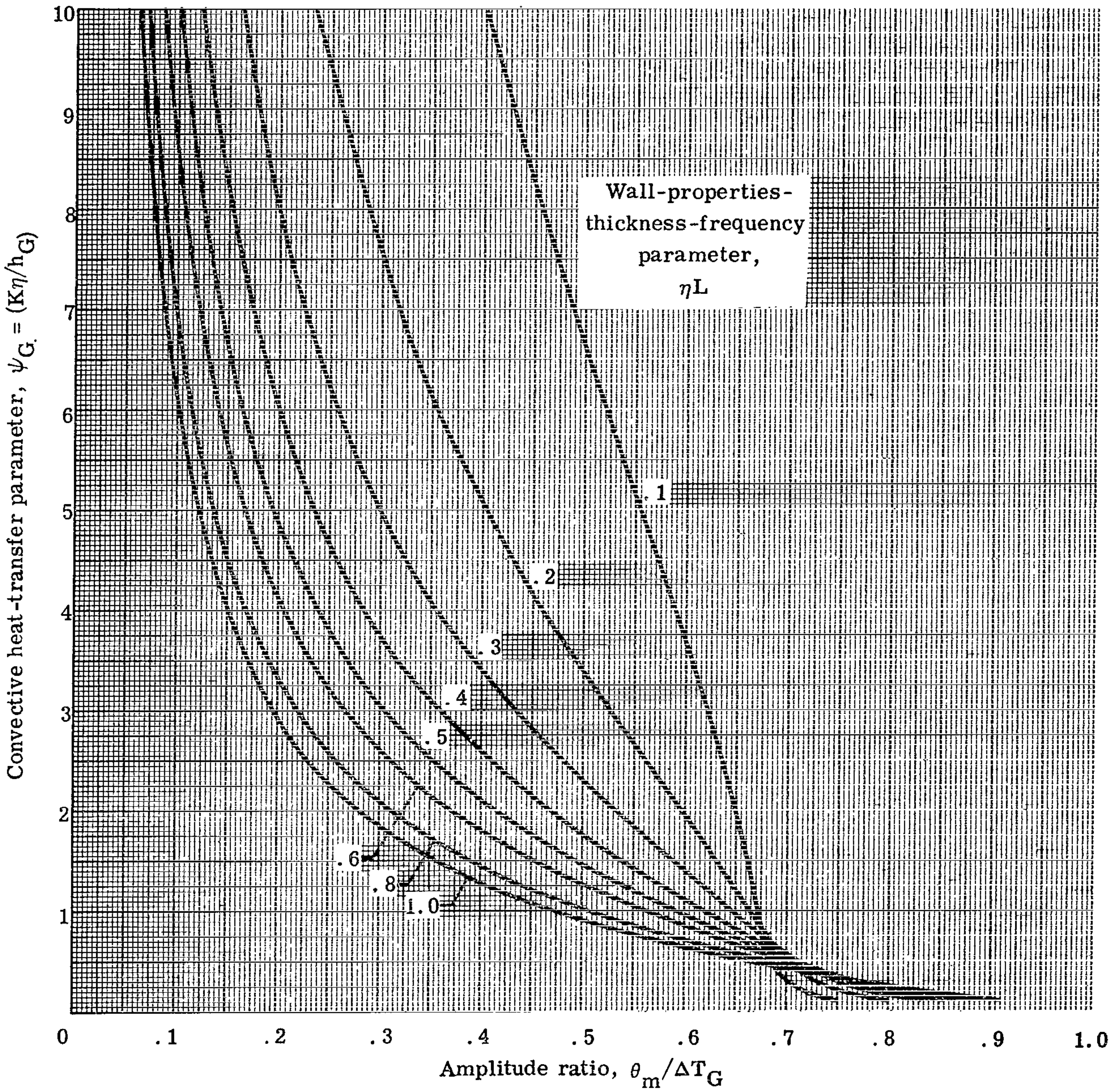
(a) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficient (insulated wall), 0.

Figure 3. - Convective heat-transfer parameter as function of wall temperature amplitude ratio.



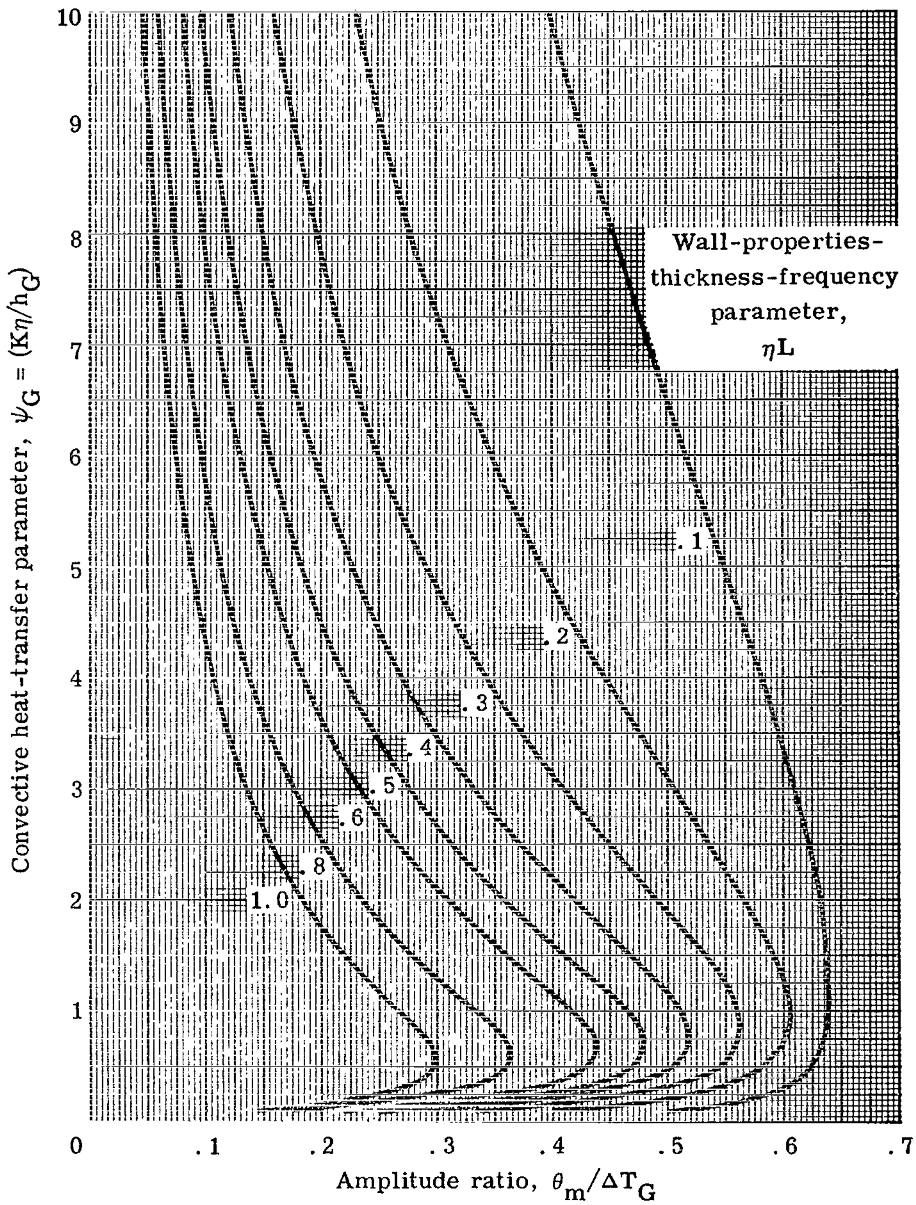
(b) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 0 (insulated wall).

Figure 3. - Continued.



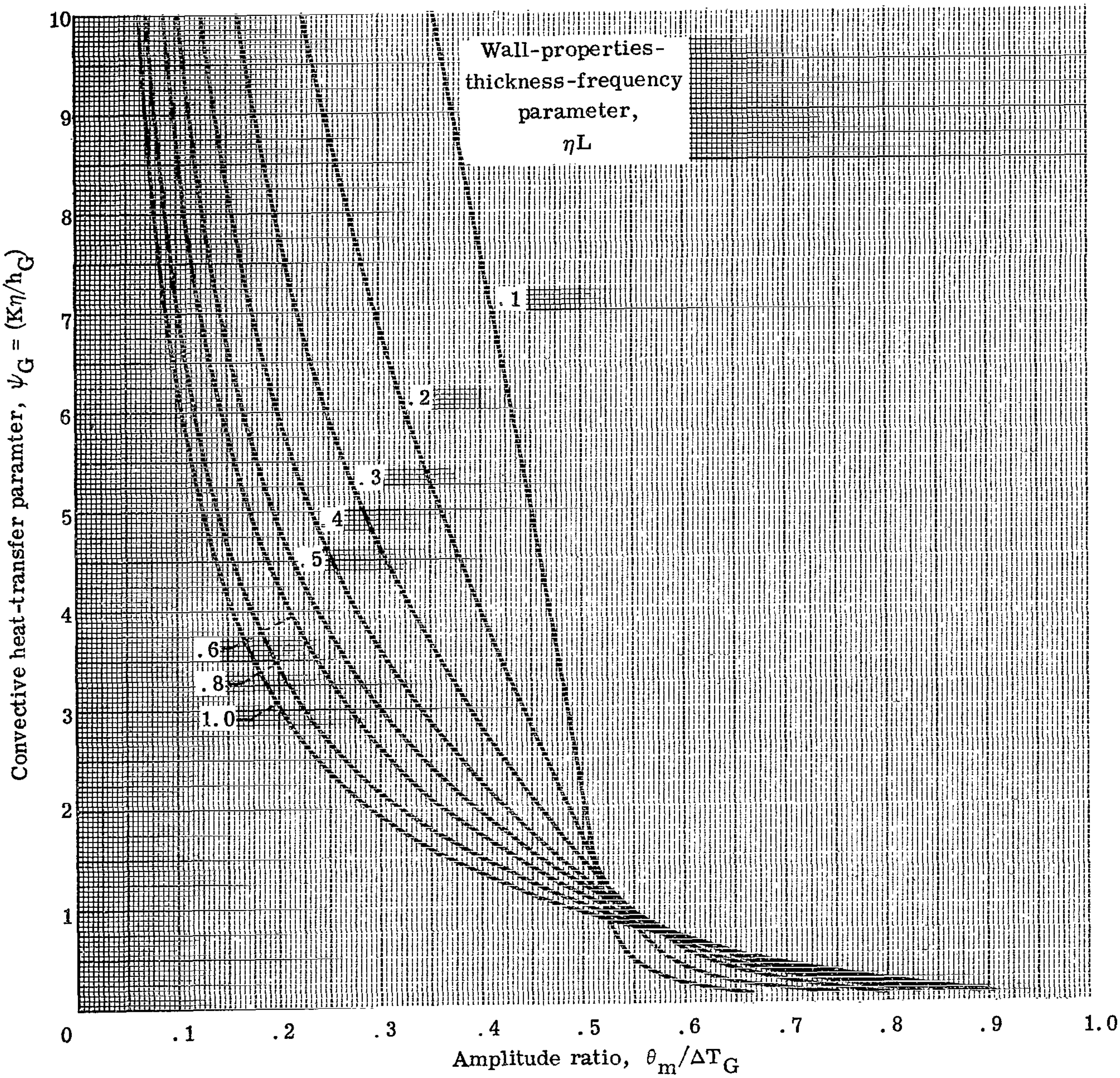
(c) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 0.5.

Figure 3. - Continued.



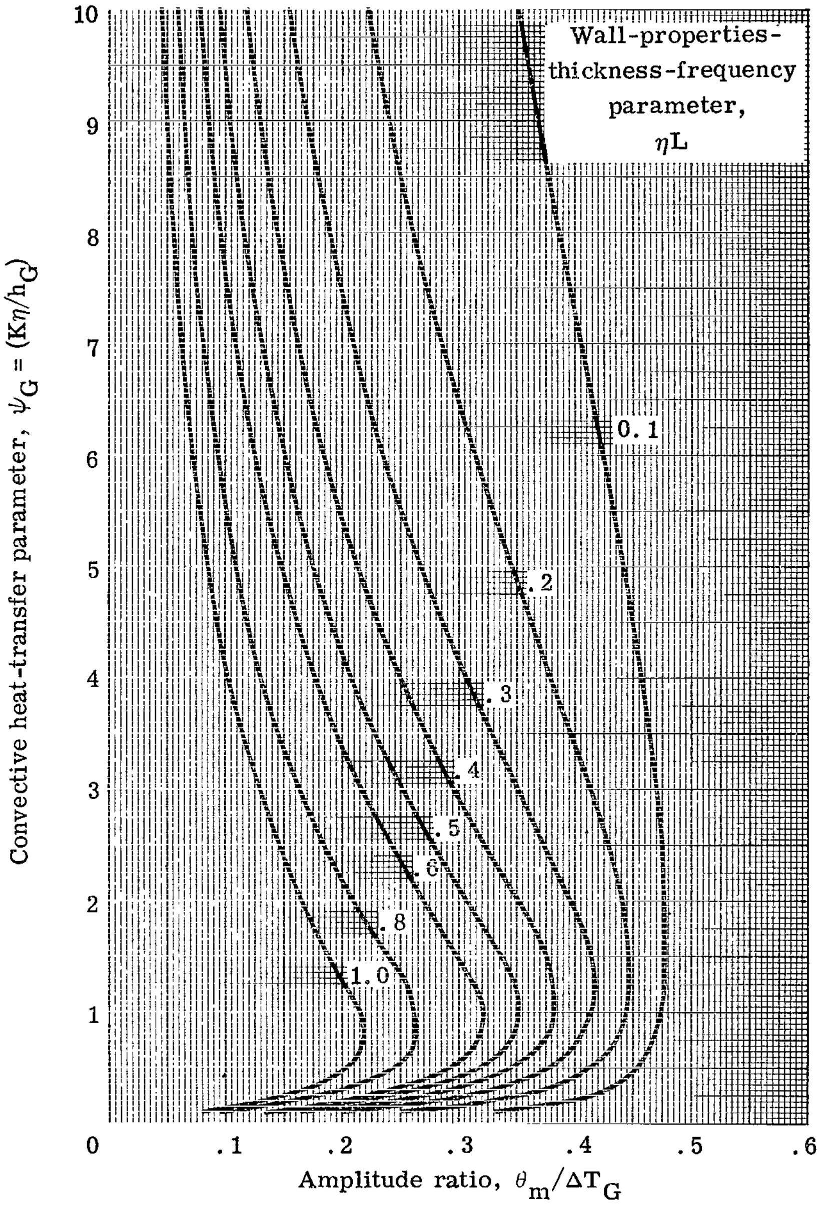
(d) Wall temperature measurement location, x/L = 1.0; ratio of convective heat-transfer coefficients, 0.5.

Figure 3. - Continued.



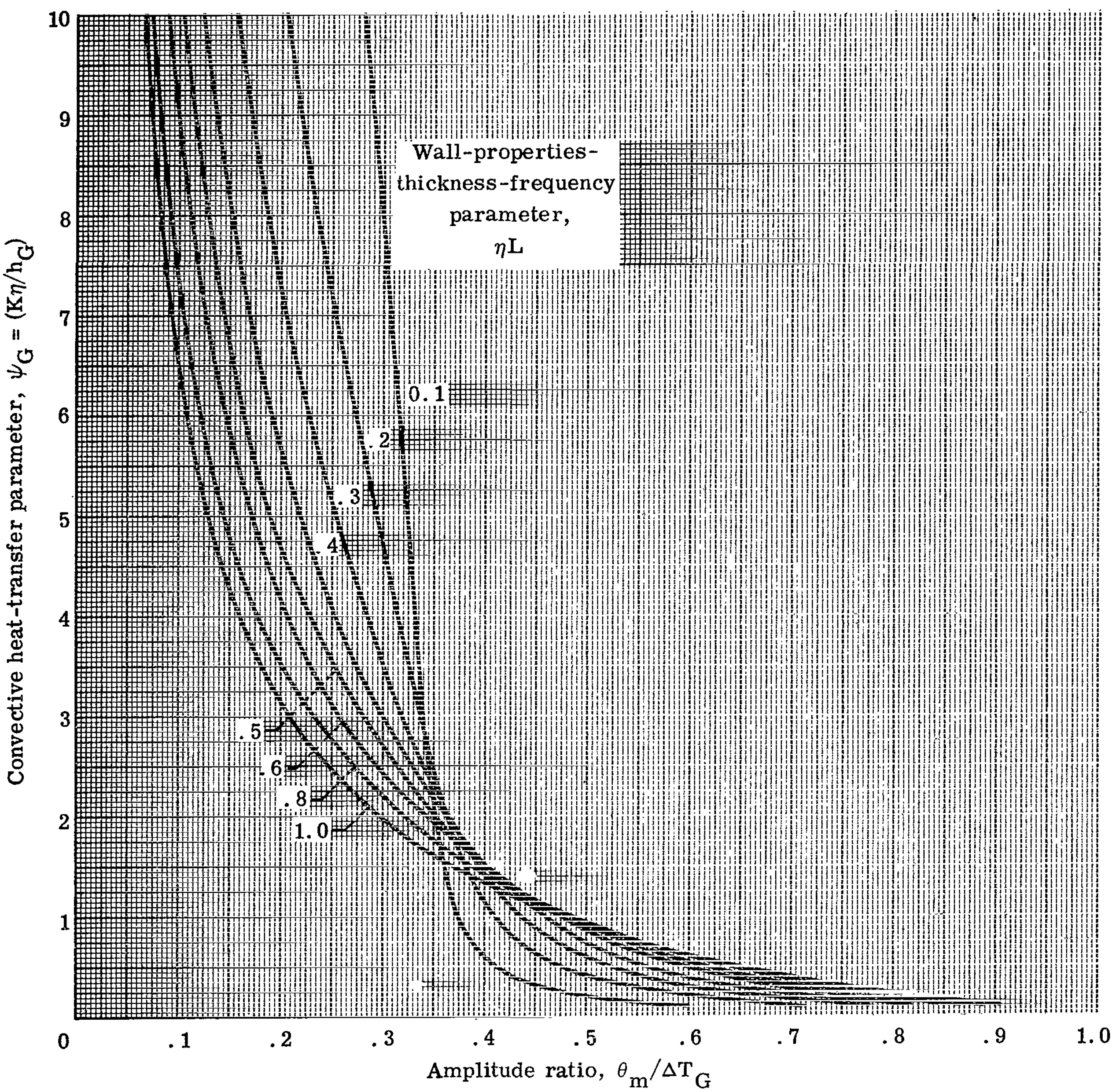
(e) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 1.0.

Figure 3. - Continued.



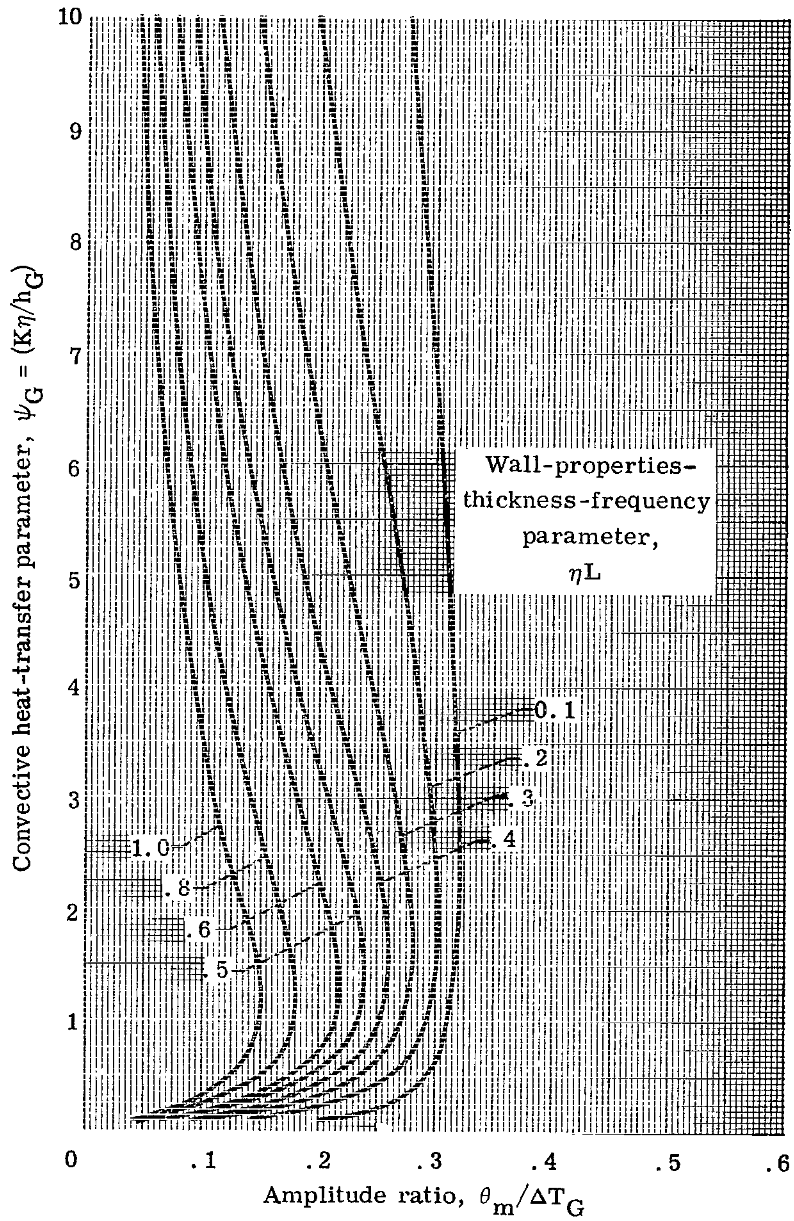
(f) Wall temperature measurement location, x/L = 1.0; ratio of convective heat-transfer coefficients, 1.0.

Figure 3. - Continued.



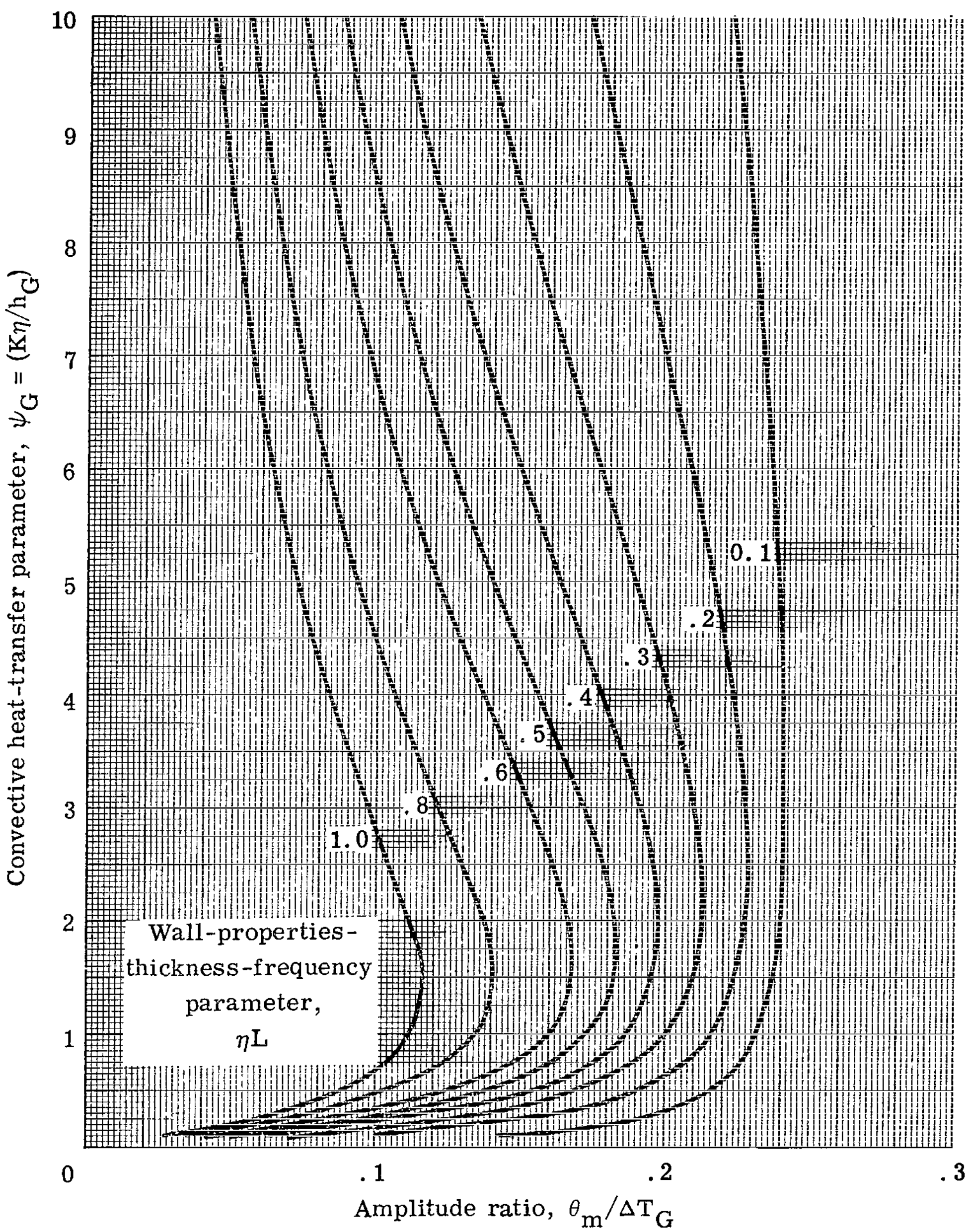
(g) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients.

Figure 3. - Continued.



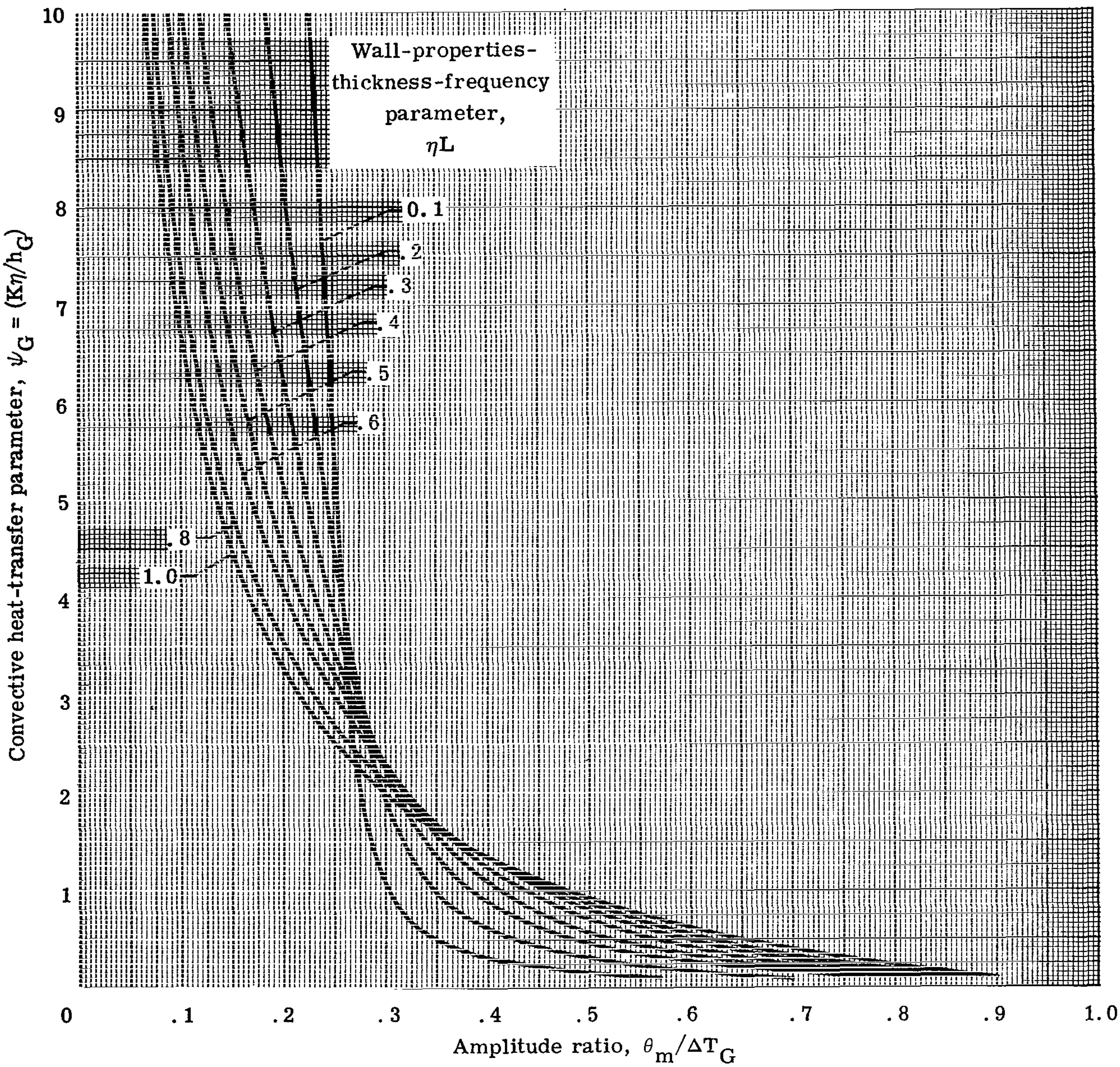
(h) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 2.0.

Figure 3. - Continued.



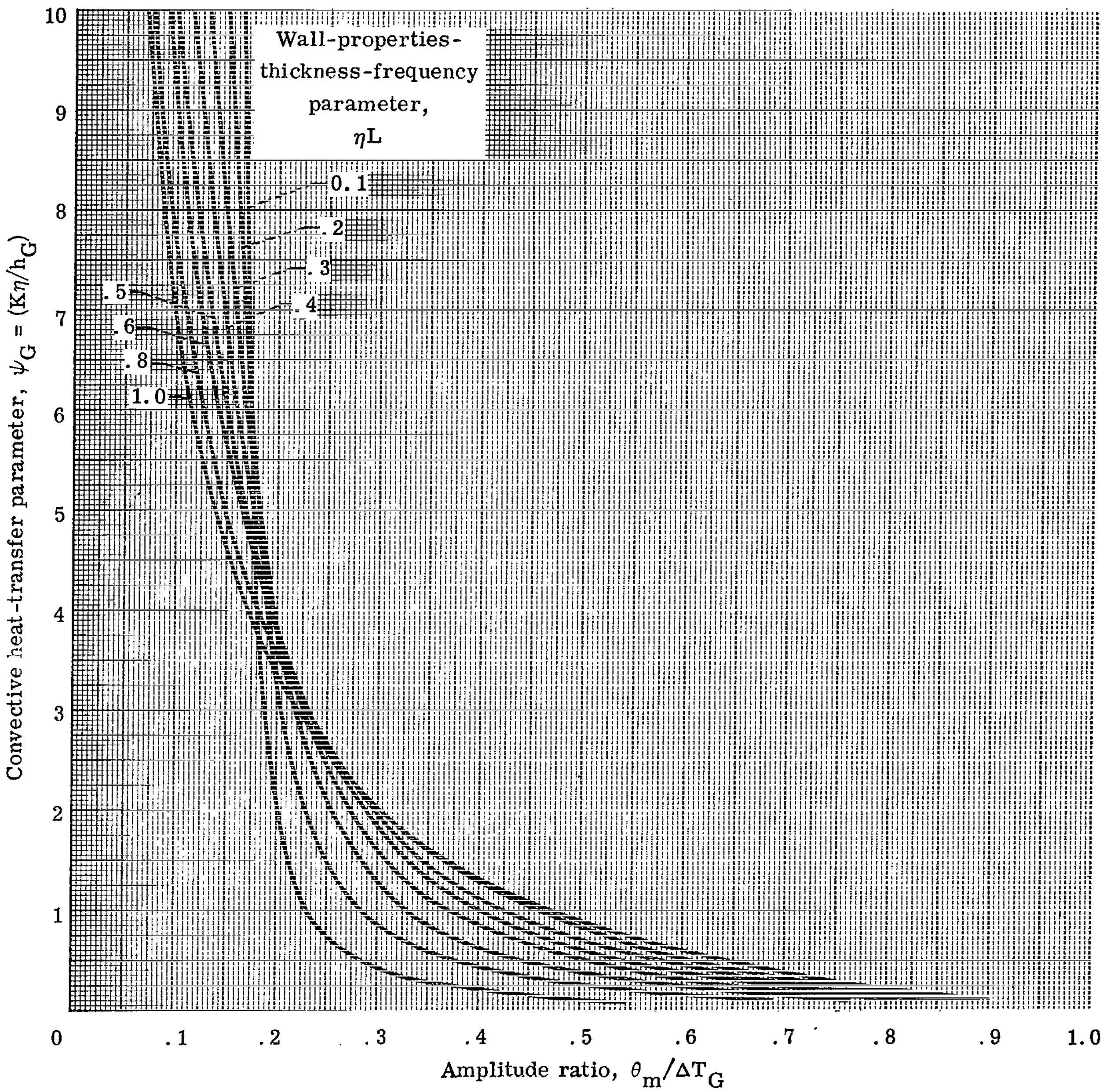
(i) Wall temperature measurement location, x/L = 1.0; ratio of convective heat-transfer coefficients, 3.0.

Figure 3. - Continued.



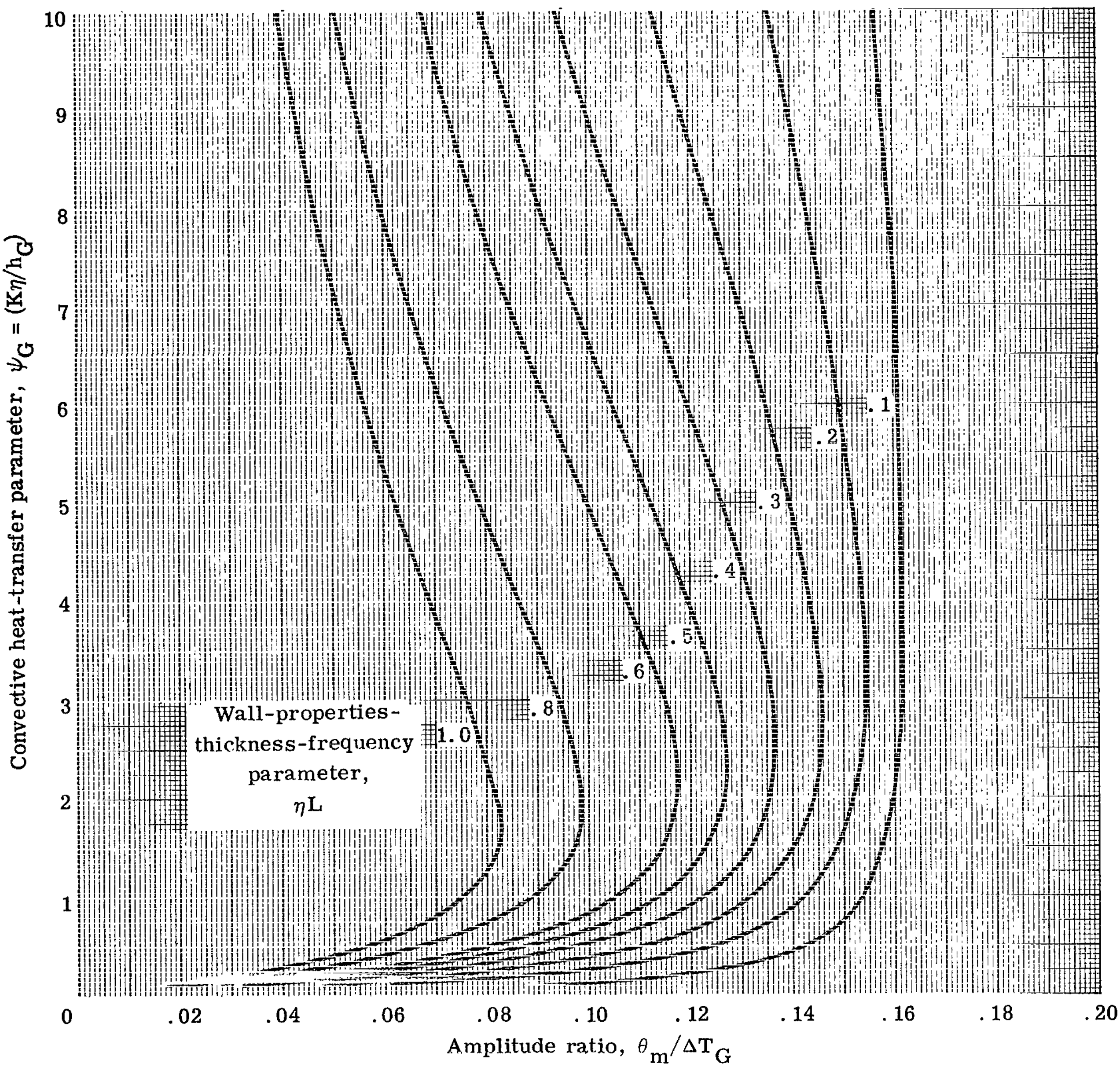
(j) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 3.0.

Figure 3. - Continued.



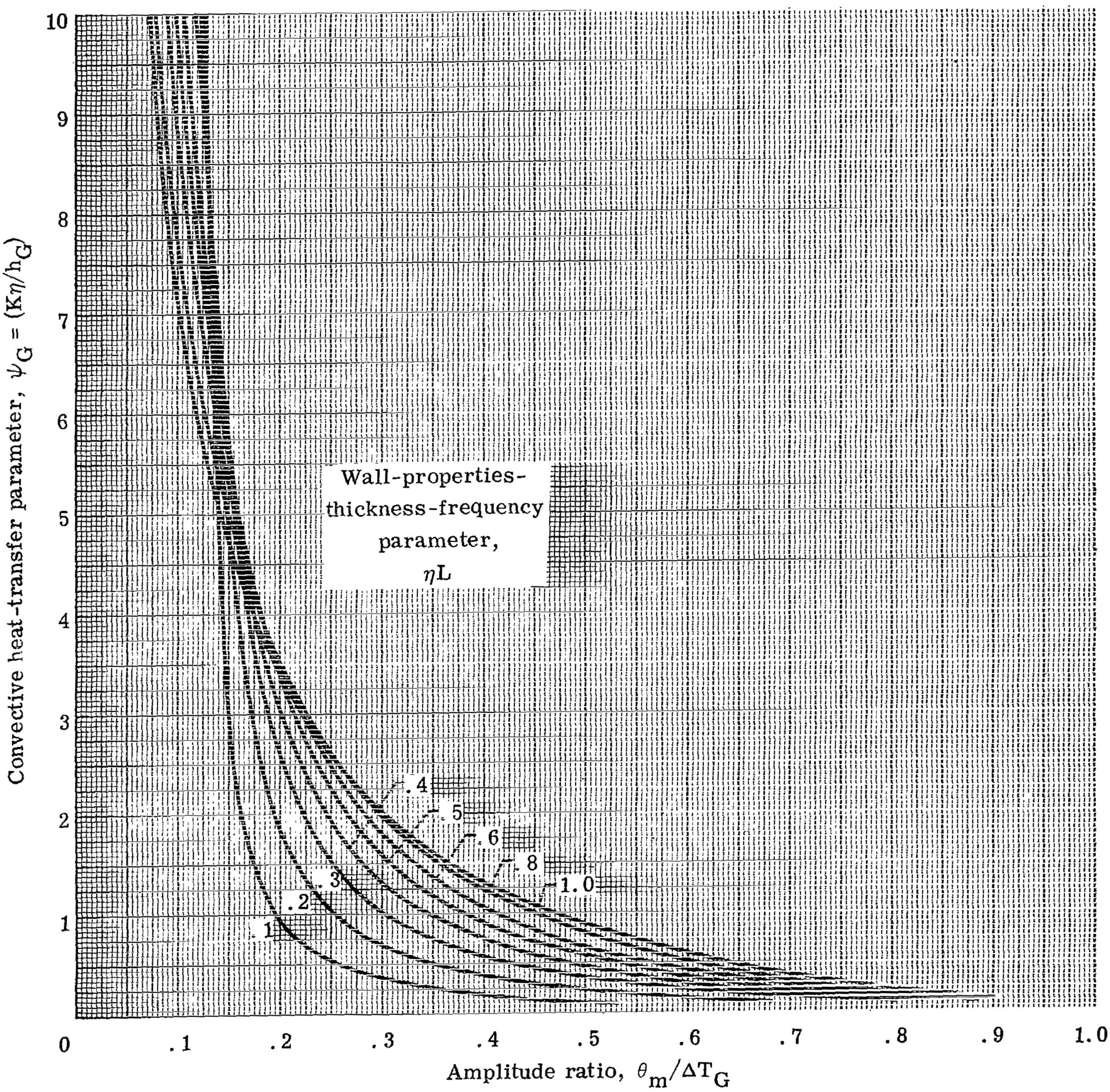
(k) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 5.0.

Figure 3. - Continued.



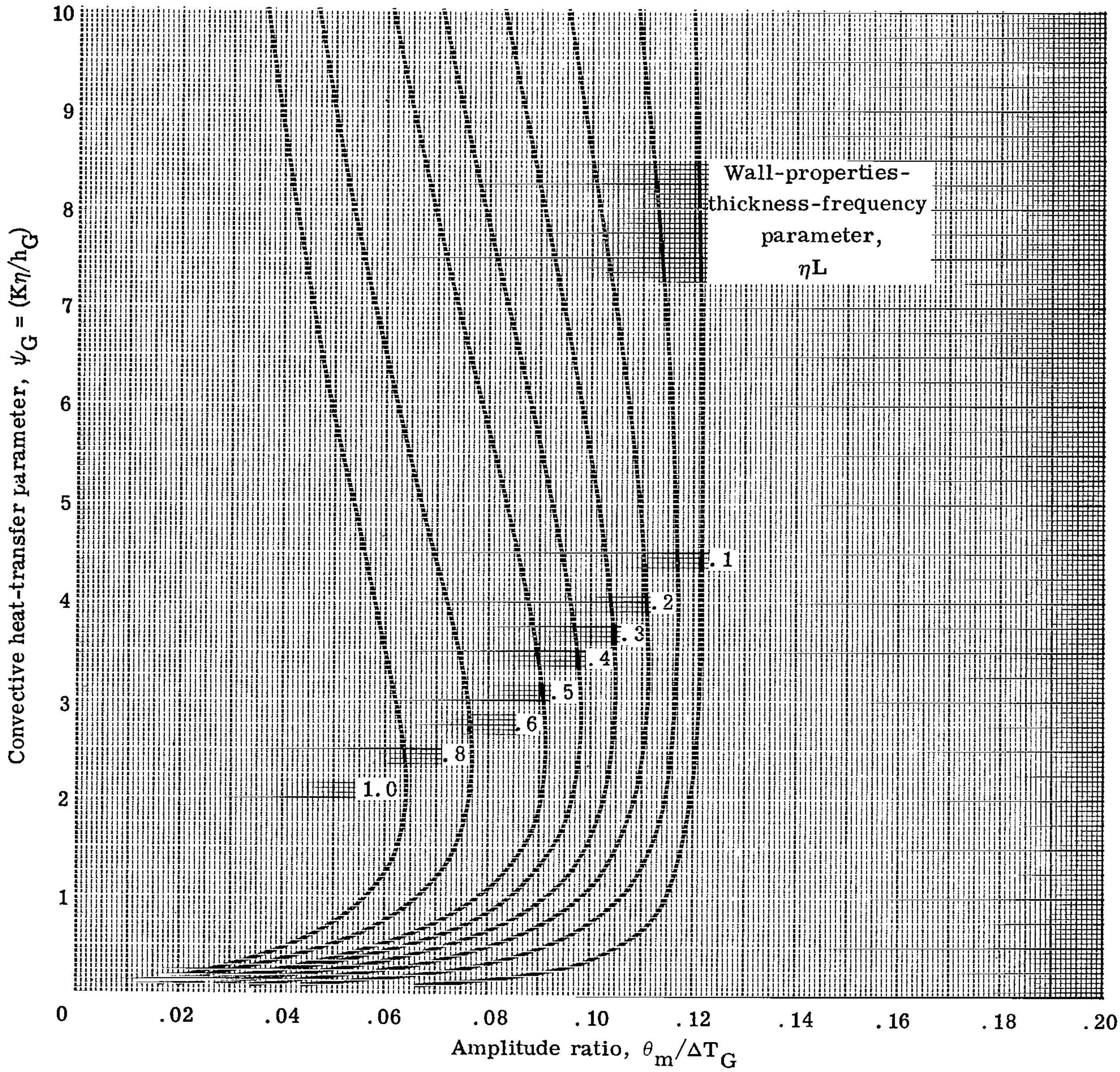
(l) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 5.0.

Figure 3. - Continued.



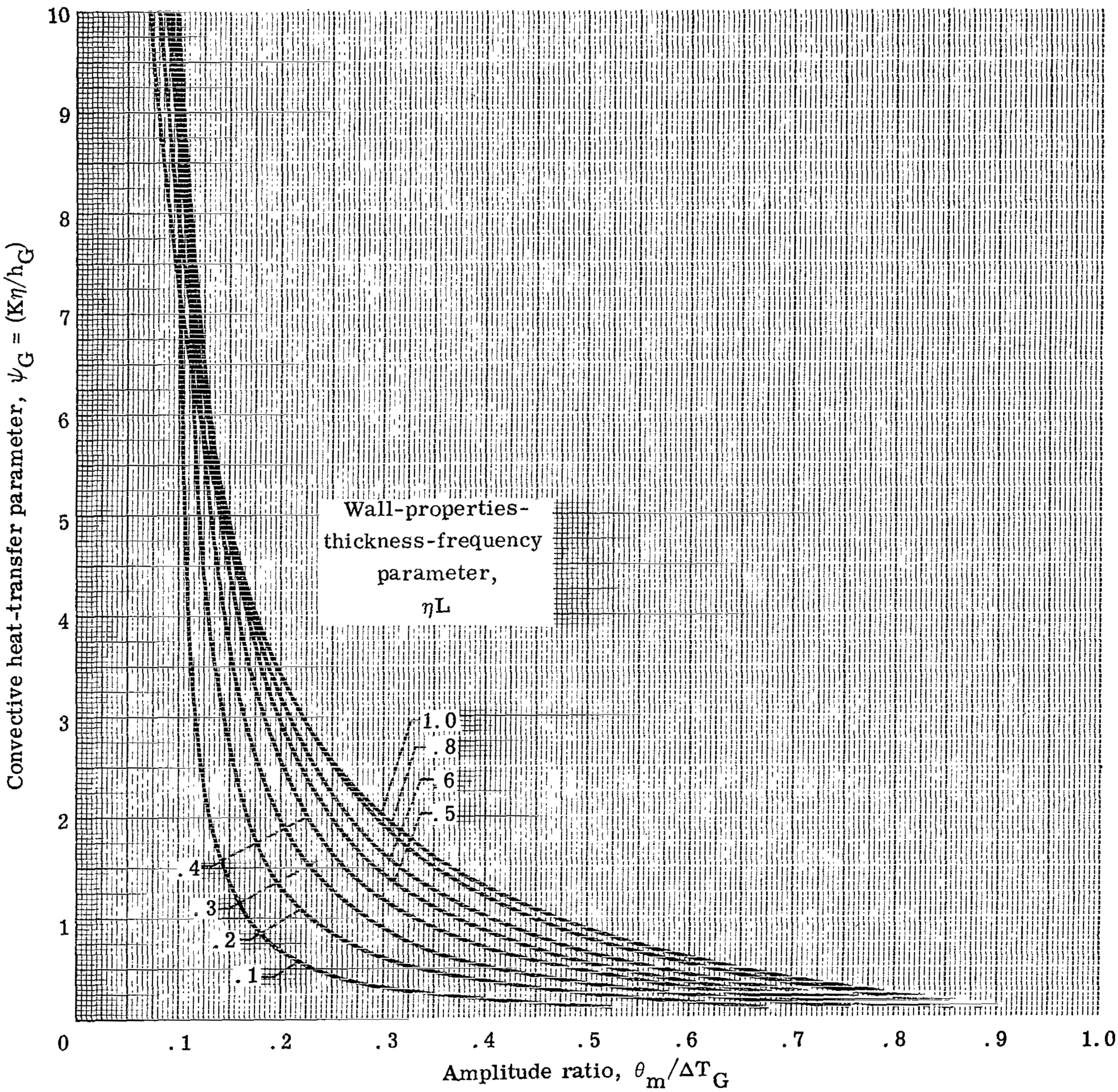
(m) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 7.0.

Figure 3. - Continued.



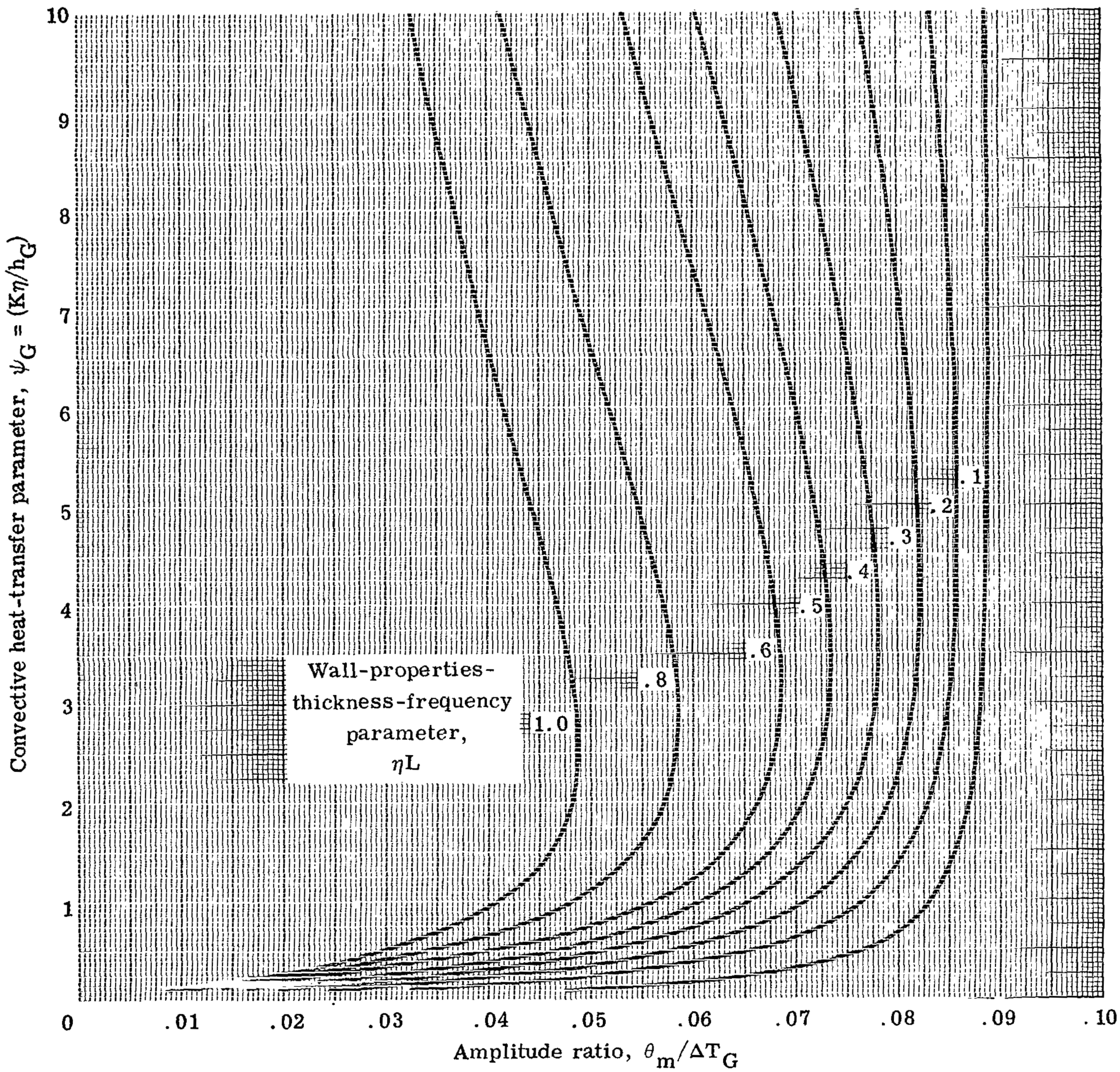
(n) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 7.0.

Figure 3. - Continued.



(o) Wall temperature measurement location, $x/L = 0$; ratio of convective heat-transfer coefficients, 10.0.

Figure 3. - Continued.



(p) Wall temperature measurement location, $x/L = 1.0$; ratio of convective heat-transfer coefficients, 10.0.

Figure 3. - Concluded.

(2) The frequency of the temperature oscillation, density, specific heat, thermal conductivity, and wall thickness are used to calculate the value of ηL. This determines which curve is to be used on the selected chart.

(3) The phase lag (in deg) of the wall temperature behind the forced fluid temperature is determined experimentally from simultaneous oscillograph traces of these temperatures. Entering the selected chart with the phase lag angle on the appropriate ηL curve gives a value for the convective heat-transfer parameter ψ_G.

(4) The convective heat-transfer coefficient on the hot-gas side is calculated by substituting ψ_G from step (3), the frequency of the temperature oscillation, and the wall properties into equation (7).

(5) The convective heat-transfer coefficient on the coolant side is calculated using either equation (8) or more simply from: $h_c = Rh_G$.

Obtaining the correct value of R in figures 2 and 3 may present some difficulty, since only one wall temperature is assumed to be measured and since equation (9) calls for both $\overline{T}(0)$ and $\overline{T}(L)$. This difficulty, however, may be circumvented as explained as follows.

If the temperature differences in equation (9) are large enough that the temperature drop across the wall does not significantly affect the ratio R, the required relation between ψ_G and ψ_c is given by equations (8) and (9) by assuming that $\overline{T}(0) = \overline{T}(L)$

If R is affected significantly in equation (9) by the wall temperatures $\overline{T}(0)$ and $\overline{T}(L)$ not being equal, an iternative process may be used. The initial value for R is calculated from equation (9) assuming that $\overline{T}(0)$ equals $\overline{T}(L)$. Resulting values for h_G and h_c will allow a heat flux q to be calculated using the measured wall temperature (assumed for this illustration to be at $x = 0$, although any known location in the wall will also be soluble) and the following equation:

$$q = h_G(\overline{T}_G - \overline{T}(0)) \tag{10}$$

The temperature drop through the wall is given by

$$\overline{T}(0) - \overline{T}(L) = \frac{qL}{K} \tag{11}$$

so that

$$\overline{T}(L) = \overline{T}(0) - \frac{qL}{K} \tag{12}$$

Use of the new value of $\overline{\overline{T}}(L)$ in equation (9) gives an improved value for R, which, in turn, can be used in the transient solution to obtain improved values of heat-transfer coefficients. The process is repeated until the heat-transfer coefficients no longer change significantly. It has been found that between three to six interations will match the heat convected to the wall to that which is conducted through the wall and convected from the wall. The agreement is within 1 percent.

Since charts cannot fit exactly the conditions of an experiment without interpolations, equation (B26) has been rearranged as shown in appendix B (eq. (B51)) to give a more direct solution for ψ_c. This equation involves the convective heat-transfer coefficient ratio R, along with ηL, x/L and the phase lag angle φ, and it can be solved using the classic cubic equation method. The correct choice of root for equation (B51) is the positive, real root.

Equation (B27) has not been rearranged to give ψ_G or ψ_c in terms of the amplitude ratio, since the amplitude ratio requires higher frequencies to get measurable amplitude ratio changes than the phase lag equation. Also, using the phase lag eliminates the need for having the absolute value of ΔT_G and thereby permits the use of a trace of chamber pressure in the case of a rocket engine to replace the oscillating trace of the hot-fluid temperature. Use of the phase lag equation also eliminates the double value problem encountered with the amplitude ratio equations. However, this equation has been programmed and gives h_G and h_c as functions of amplitude ratio. Both equations (B51) and (B27) have been programmed (unpublished report by G. Aling and R. Huff) in FORTRAN IV and are operational on the IBM 7094-2/7044 direct-coupled system. The titles of the programs are JCL and BL, respectively.

Lumped Wall Properties (Case 2)

The assumptions used in the case 1 solution have been applied to the case 2 solutions with one exception. Assumption (1) has been eliminated, and in its place it has been assumed that the temperature gradient through the wall is small or that the wall temperature is measured at a point in the wall where the thermal properties are considered to be lumped (appendix C).

The governing differential equation can be written by equating the difference between the heat transferred to the wall and the heat that leaves the wall to the heat stored in the wall. In writing the differential equation which describes this condition, it is possible to account for the thermal conductivity of the wall by using a modified overall heat-transfer coefficient. The form of the differential equation is the same as that which results when the thermal resistance of the wall is neglected. Therefore, the differential equation that neglects the effects of thermal conductivity is used in the initial derivation and then the solution is modified to account for the thermal conductivity of the wall material. The differential equation used in the initial derivation (appendix C) is

$$\rho c L \frac{\partial T}{\partial \tau} = h_G(T_G - T) - h_c(T - T_c) \tag{13}$$

The hot-gas-side fluid temperature T_G is driven sinusoidally and is given by

$$T_G = \overline{T}_G + \Delta T_G \sin \omega\tau \tag{14}$$

The solution to equation (13), using Laplace transform techniques is detailed in appendix C and is given here in functional form as

$$T_1(\tau) = \underbrace{M(T_o, \overline{T}_G, T_c, \Delta T_G, \omega, \rho, c, L, h_G, h_c)\exp - \frac{h_G + h_c}{\rho c L}\tau}_{\text{Starting transient}} +$$

$$\underbrace{N(\overline{T}_G, T_c, h_G, h_c)}_{\text{Offset from conditions before time zero}} + \underbrace{\theta_1(\Delta T_G, \omega, \rho, c, L, h_G, h_c)\sin(\omega\tau - \varphi_1)}_{\text{Steady-state oscillation}}$$

where

$$\varphi_1 = \varphi(\rho, c, L, \omega, h_G, h_c)$$

Laplace transforms are used because they yield a solution that gives the wall temperature history from just before the initial gas temperature fluctuation to the time when the wall temperature reaches a steady-state (neutral) oscillation. The solution for the wall temperature is given by equation (C9). The phase lag is given by equation (C10).

By assuming that the ratio of the convective heat-transfer coefficients R can be calculated from equation (9), the equations for the heat-transfer coefficients in terms of either phase lag angle or amplitude ratio can be derived from equations (C10) and (C11), respectively. These are

$$h_G = \frac{\rho c L \omega}{(1 + R)\tan \varphi_1} \tag{15}$$

and

$$h_G = \rho c L \omega \left[\frac{1}{\left(\frac{\theta_{m1}}{\Delta T_G}\right)^2} - (1 + R)^2 \right]^{-1/2} \tag{16}$$

The coolant side coefficient from the definition of R is

$$h_c = R h_G \tag{17}$$

The time required for the decay of the initial transient of the wall temperature so that it is influenced only by the hot-gas temperature oscillations can be determined by using the first three terms of equation (C9) (starting transient). This calculation serves only as an approximation as to the time required to reach a steady-state oscillating condition. The approximation should be good when the thermal diffusivity of the wall material is high.

To account for the thermal conductivity of the wall material when using the lumped-wall-property method, equation (13) can be modified by replacing the convective heat-transfer coefficients with a modified overall coefficient of heat transfer. The details of this procedure are given in the second section of appendix C. This modification results in the following equations for the wall temperature amplitude ratio and phase lag angle

From equation (C18) the amplitude ratio is

$$\frac{\theta_{m2}}{\Delta T_G} = \frac{\cos \varphi_2}{1 + \dfrac{R\left(1 + \dfrac{x_G}{L} \dfrac{\eta L}{\psi_G}\right)}{1 + \left(1 - \dfrac{x_G}{L}\right) \dfrac{\eta L R}{\psi_G}}} \tag{18}$$

From equation (C23) the phase lag angle is

$$\varphi_2 = \tan^{-1} \frac{2\eta L \psi_G \left(1 + \dfrac{x_G}{L} \dfrac{\eta L}{\psi_G}\right)}{1 + \dfrac{R\left(1 + \dfrac{x_G}{L} \dfrac{\eta L}{\psi_G}\right)}{1 + \left(1 - \dfrac{x_G}{L}\right) \dfrac{\eta L R}{\psi_G}}} \tag{19}$$

Equations (18) and (19) can be rearranged so that the convective heat-transfer parameter ψ_G can be calculated directly. Equation (18) becomes

$$\psi_{G2} = \frac{\eta LR\left[\left(1 - \frac{x_G}{L}\right)\cos\varphi_2 - \frac{\theta_{m2}}{\Delta T_G}\right]}{\frac{\theta_{m2}}{\Delta T_G}(1 + R) - \cos\varphi_2} \tag{20}$$

and equation (19) becomes

$$\psi_{G2} = \frac{(1 + R)}{4\eta L}\tan\varphi_2 - \frac{\eta L}{2}\left[\frac{x_G}{2} + \left(1 - \frac{x_G}{L}\right)R\right] \pm \frac{1}{2}\left(\left\{\eta L\left[\frac{x_G}{L} + \left(1 - \frac{x_G}{L}\right)R\right] - \frac{(1+R)}{2\eta L}\tan\varphi_2\right\}^2 - 4\left[\frac{x_G}{L}\left(1 - \frac{x_G}{L}\right)(\eta L)^2 R - \frac{R}{2}\tan\varphi_2\right]\right)^{1/2} \tag{21}$$

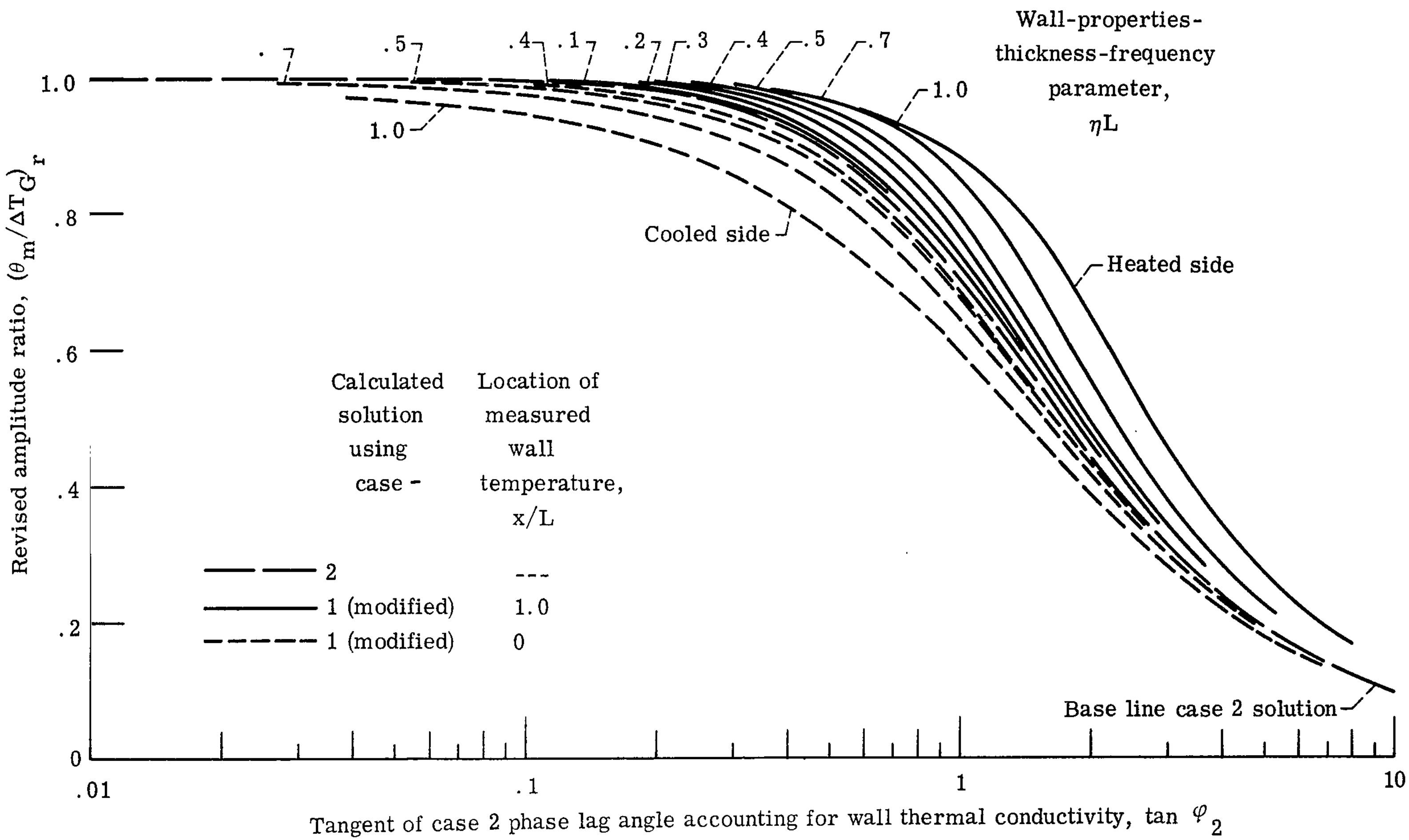
COMPARISON OF DISTRIBUTED (CASE 1) AND LUMPED (CASE 2) SOLUTIONS

Generally, case 2 solution not accounting for conductivity is preferred because of its mathematical simplicity; however, large errors in the convective heat-transfer coefficients may be introduced by using the case 2 solution.

The best way to determine whether the case 2 solution can be used in a particular experiment is to compare the heat-transfer coefficients calculated using both the case 1 and case 2 solutions over the expected range of operating conditions. If the difference is negligible, the case 2 solution would, for simplicity purposes, be used.

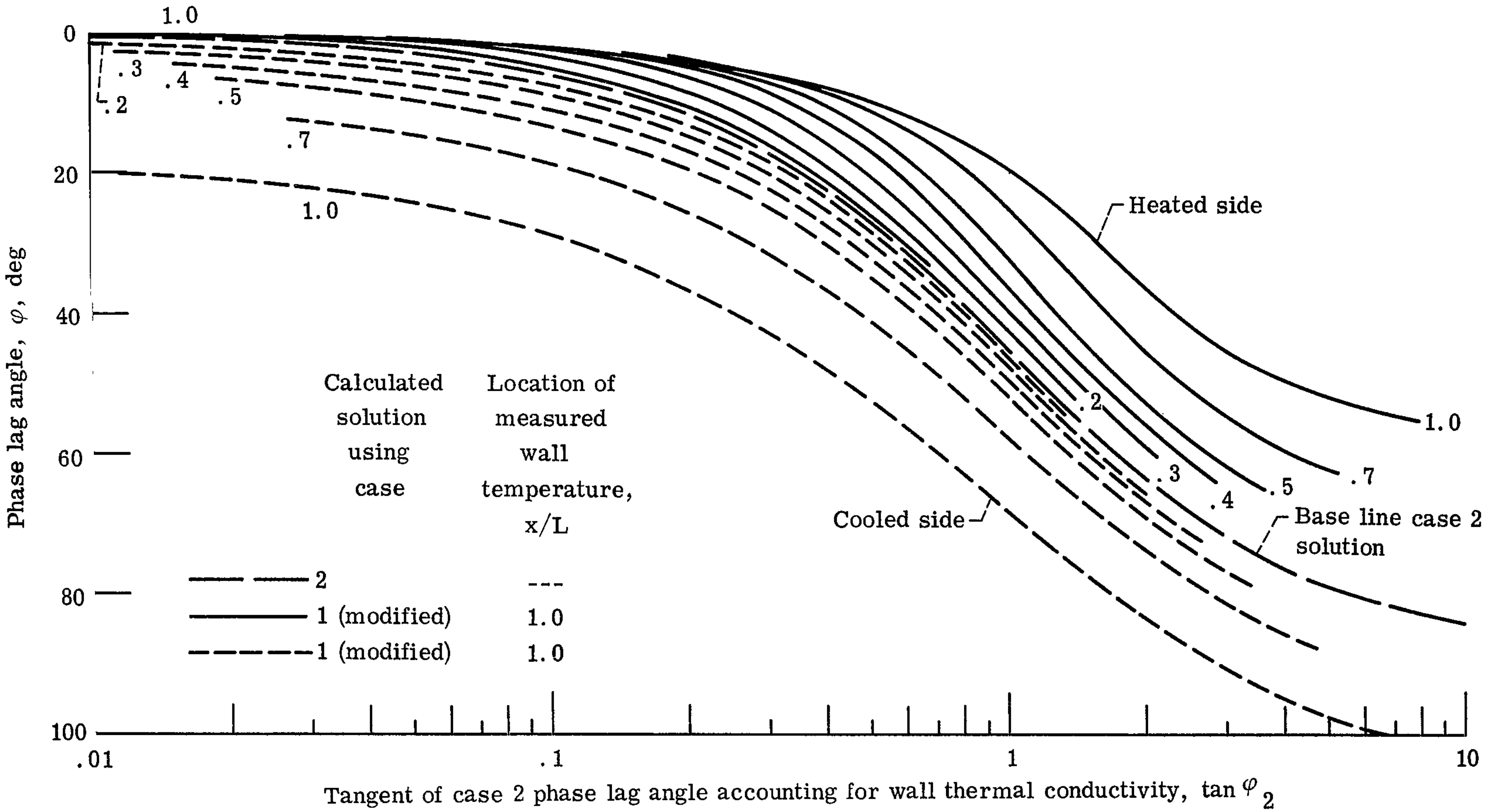
To compare case 1 with case 2, a plot of the revised amplitude ratio (fig. 4(a)) and phase lag angle (fig. 4(b)) versus the tangent of the case 2 phase lag angle is shown in figure 4. The case 2 solution provides a base line, for comparison purposes on both plots, that does not shift with changing ηL, x/L, or R values. The reason for the nonshifting base line in figure 4(a) is that the revised amplitude ratio is defined as

$$\frac{\theta_{m2}}{\Delta T_G} = \left[1 + \frac{R\left(1 + \frac{x}{L}\frac{\eta L}{\psi_G}\right)}{1 + \left(1 - \frac{x}{L}\right)\frac{\eta LR}{\psi_G}}\right]\frac{\theta_{m2}}{\Delta T_G} \tag{22}$$



(a) Wall temperature amplitude ratio.

Figure 4. - Comparison of case 1 and case 2 solutions; ratio of convective heat-transfer coefficients, 5.0.



(b) Phase lag angle.

Figure 4. - Concluded.

which, from equation (18), is equal to $\cos \varphi_2$. Plotting the $\cos \varphi_2$ versus the $\tan \varphi_2$ gives the base line curve shown in figure 4(a), which is not a function of ηL, x/L, or R. The reason for the nonshifting base line in figure 4(b) is, from inspection of equation (19), that the base line is merely a plot of the phase lag angle versus the tangent of the phase lag angle. The amplitude ratio (fig. 4(a)) and the phase lag angle (fig. 4(b)) are presented in this manner so the reader may relate these solutions to their electromechanical analogies, which are often plotted this way. The base lines are the same as those which result from analysis of an electrical series resistance-capacitance circuit driven by a sinusoidal voltage.

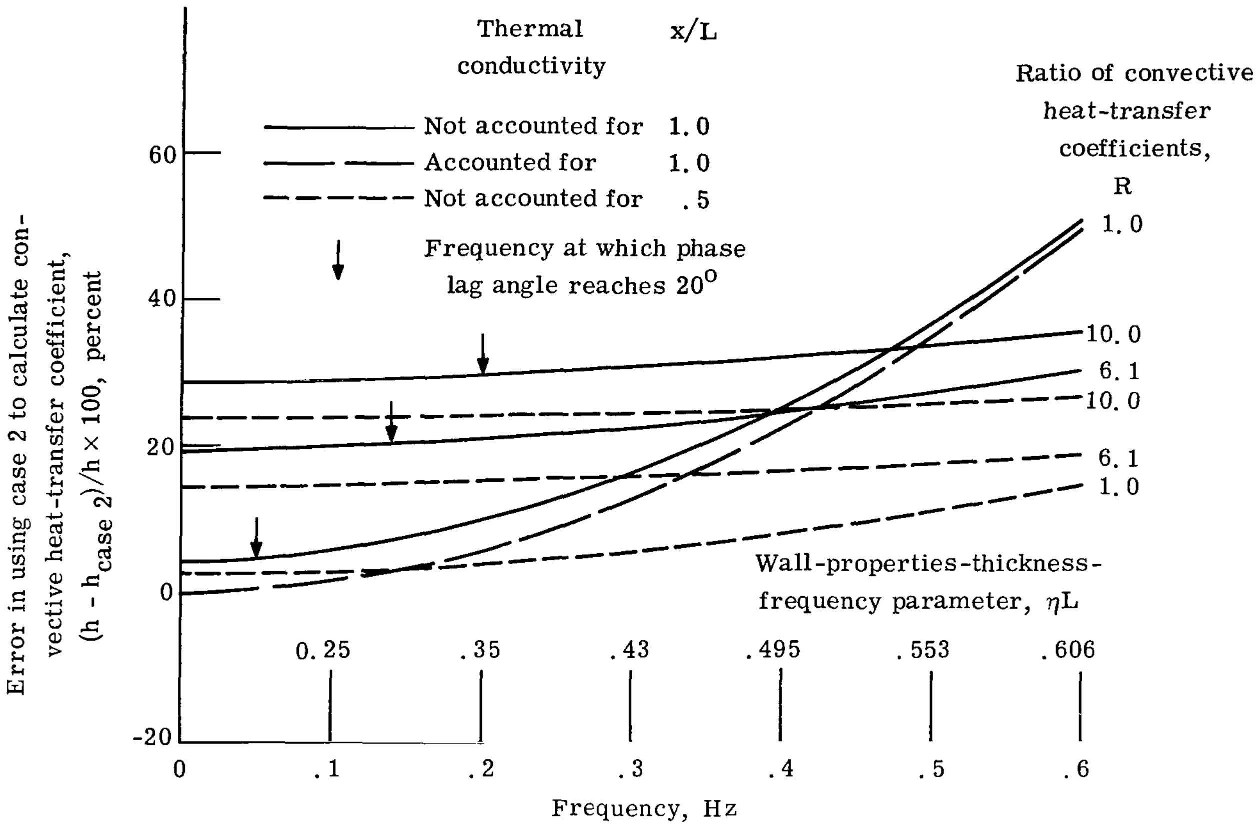
In order to compare the amplitude ratio for the case 1 solution with the base line, the case 1 solution for amplitude ratio $\theta_m/\Delta T_G$ (eq. (B27)) was multiplied by the bracketed quantity in equation (22), and this revised $(\theta_m/\Delta T_G)_r$ is plotted on figure 4(a) versus the $\tan \varphi_2$ calculated using equation (19). In this way, the case 2 solution accounting for thermal conductivity can be used when the case 1 plots fall on or near the base line curves of case 2.

Similarly, the phase lag angles calculated using case 1 (eq. (B26)) may be compared in figure 4(b) to the base line of case 2. The case 1 phase lag angles are plotted versus the $\tan \varphi_2$, calculated using equation (19). Again, when the curves fall close to or on the base line the case 2 solution accounting for the thermal conductivity can be used in place of case 1.

The set of curves shown in figure 4 are calculated for $R = 5.0$ and $x/L = 0$ and 1.0. These conditions only affect case 1 curves, because the base-line curves are not influenced by these parameters. The general shape of the curves in figure 4 is the same for values of R between 0.5 and 10.0. Inspection of figures 4(a) and (b) shows that small values of ηL tend to allow the use of case 2 solutions.

To show the order of magnitude of the errors in convective heat-transfer coefficients that could result by using the case 2 solution instead of case 1, two examples will be presented (based on phase lag angle calculations). The example shown as a function of frequency in figure 5 assumes a rocket engine having a stainless-steel wall 0.012 inch (0.3048 mm) thick and a hot-gas-side heat-transfer coefficient of 0.0022 Btu per square inch per $^{\circ}$R (6.472 kW/(m^2)(K)).

For the rocket engine example, an R value of 6.1 is considered representative; however, a range of R values are presented. For these conditions and an $x/L = 1.0$ (fig. 5(a)), errors from 30 percent at 6 hertz ($\eta L = 0.606$) to 20 percent at 1.0 hertz



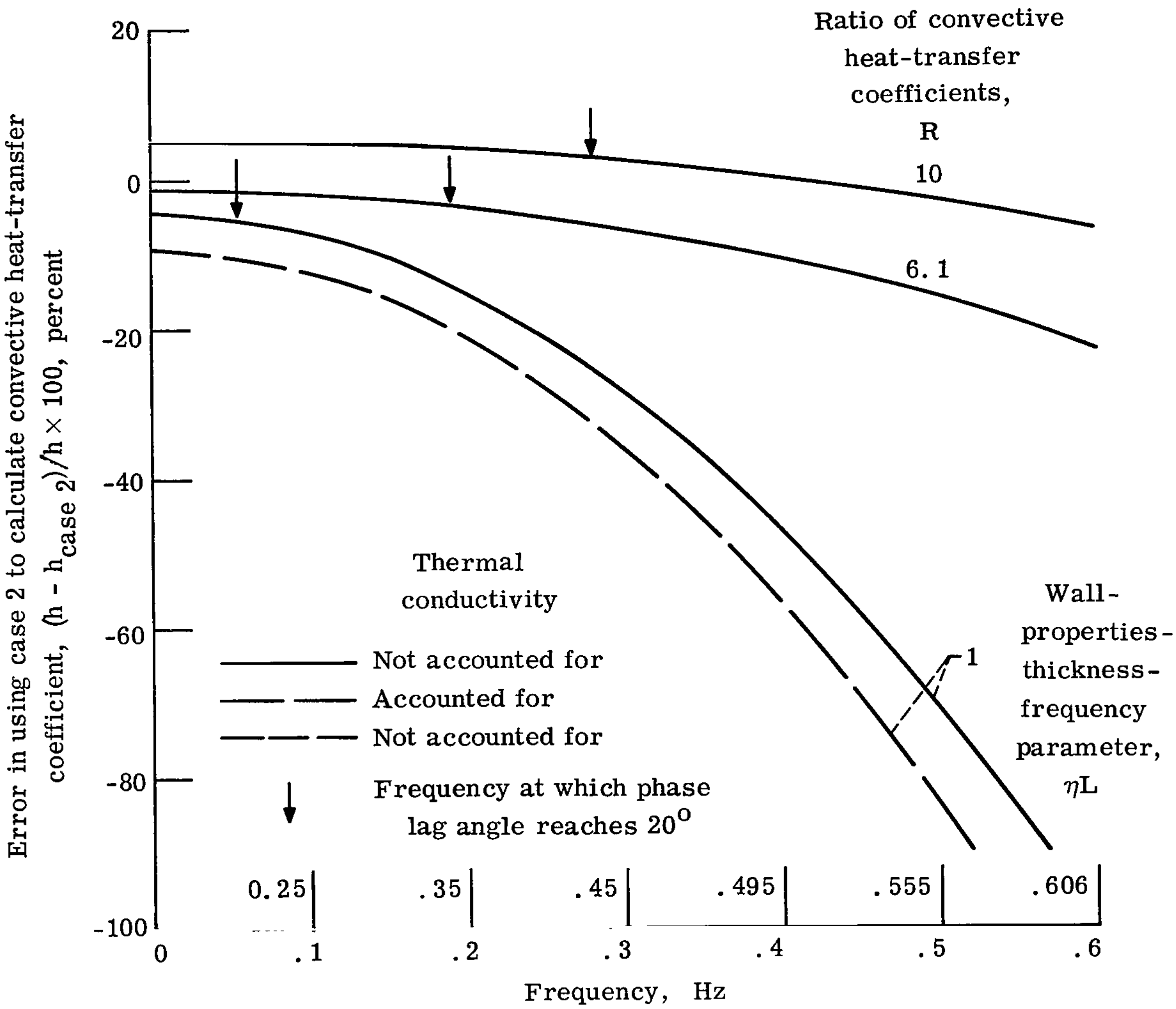
(a) Wall temperature maasurement location, x/L = 1.0 and 0.5.

Figure 5. - Percent error in convective heat-transfer coefficients using lumped wall properties for rocket engine example. Assumed wall conditions: density, 0.286 pound mass per cubic inch (7.92 g/m^3); specific heat, 0.1358 Btu per pound mass per $^{\circ}R$ (1.5686×10^3 J/(kg)(K)); thermal conductivity, 0.287×10^{-3} Btu per inch per second per $^{\circ}R$ (21.46 J/(m)(sec)(K)); thickness, 0.012 inch (0.3048×10^{-3} m); gas-side heat-transfer coefficient, 0.0022 Btu per square inch per second per $^{\circ}R$ (6.472 $kW/(m^2)(K)$); $h_c = R\, h_G$.

(ηL = 0.25) are shown to be possible. At x/L = 0 (fig. 5(b)), errors are reduced to -22 percent at 6 hertz and -2 percent at 1.0 hertz.

The percent errors are based on the assumed value of h. The case 1 solution (eq. (B26)) was used to calculate the phase lag angle; this angle was then used in the case 2 solution (eq. (15), neglecting the thermal conductivity of the wall) to calculate the heat-transfer coefficient.

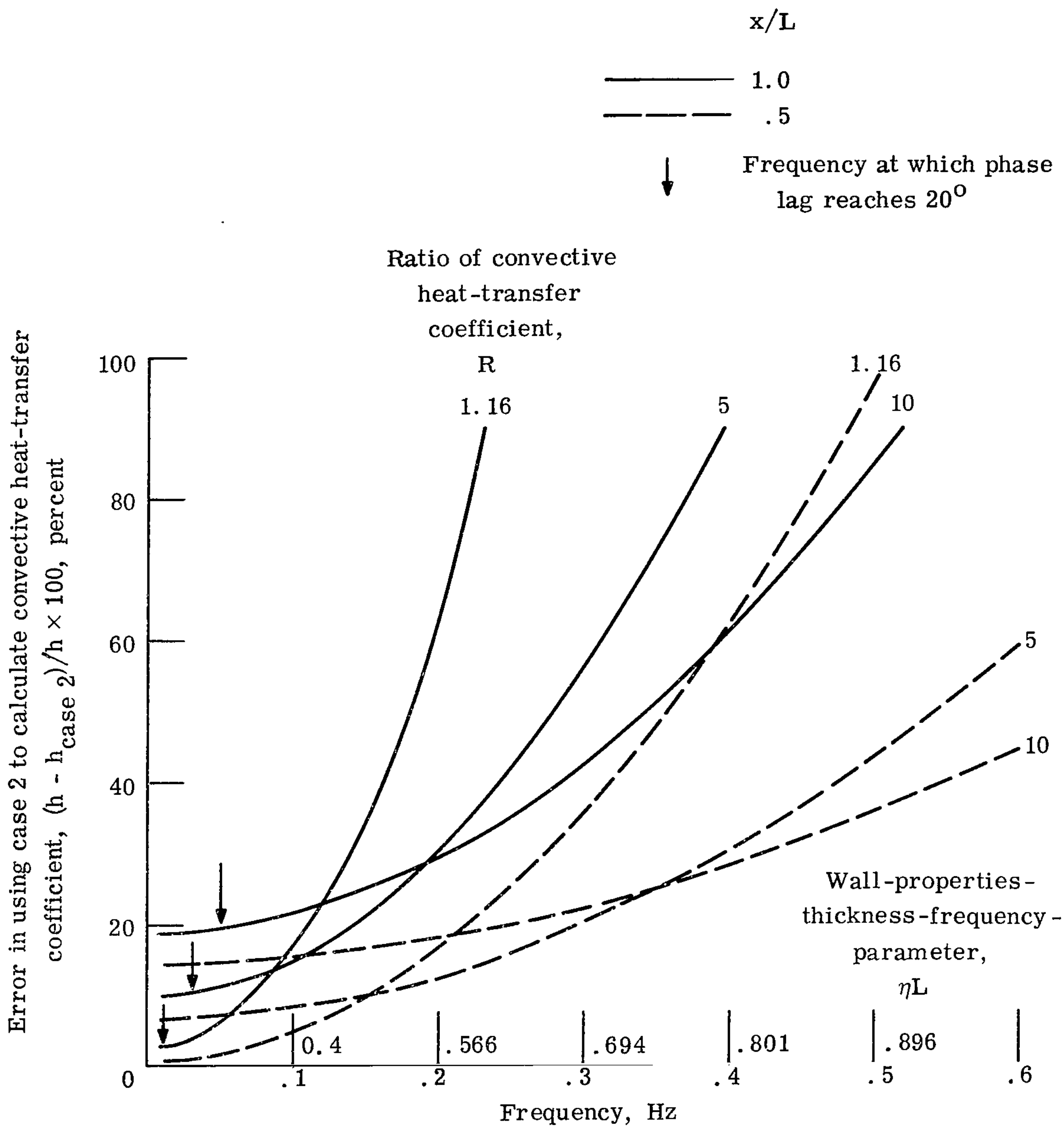
The second example assumes a much lower heat-transfer coefficient on the hot-gas



(b) Wall temperature measurement location, $x/L = 0$.

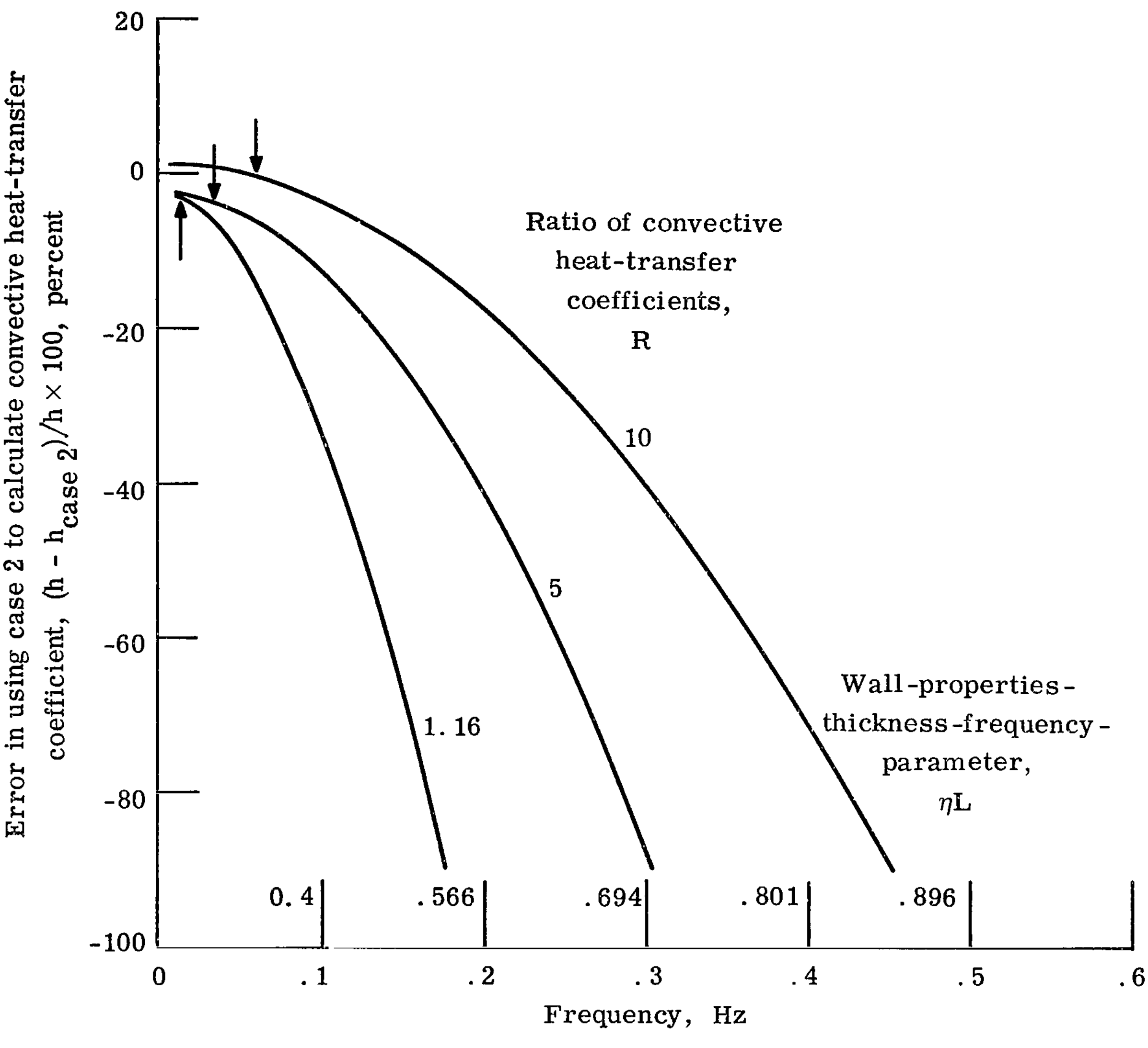
Figure 5. - Concluded.

side (by a factor of 10), considered to be representative of a heat exchanger. Figure 6 shows coefficient errors for the heat exchanger example. This example assumes a stainless-steel wall 0.06 inch (1.524 mm) thick, and a hot-gas-side heat-transfer coefficient of 0.00021 Btu per square inch per second per $^{\circ}R$ (0.61782 kW/(m^2)(K)). An R value of 1.16 was considered representative of the heat exchanger case; again however, a range of R values are presented. The coefficient errors were calculated using the same procedure followed for the rocket engine example. Under these conditions for $x/L = 1.0$ (fig. 6(a)), the errors were as large as 62 percent at 0.2 hertz ($\eta L = 0.566$)



(a) Wall temperature measurment location, x/L = 1.0 and 0.5.

Figure 6. - Percent error in convective heat-transfer coefficients using lumped wall properties for heat exchanger example. Assumed wall conditions: density, 0.286 pound mass per cubic inch (7.92 g/m^3); specific heat, 0.120 Btu per pound mass per $^{\circ}$R (0.5024$\times 10^3$ J/kg)(K)); thermal conductivity, 0.2411$\times 10^3$ Btu per inch per second per $^{\circ}$R (18.02 J/(m)(sec)(K)); thickness, 0.06 inch (1.524 mm); hot-gas side convective heat-transfer coefficient, 0.00021 Btu per square inch per second per $^{\circ}$R (0.6178 kW/(m^2)(K)); $h_c = R\, h_G$.



(b) Wall temperature measurement location, x/L = 0.

Figure 6. - Concluded.

and as small as 3 percent of 0.01 hertz ($\eta L = 0.1267$). For $x/L = 0$ (fig. 6(b)), the errors exceed -100 percent at 0.2 hertz ($\eta L = 0.0566$) but are as small as -3 percent at 0.01 hertz ($\eta L = 0.1267$).

Both examples show that decreasing the frequency (also ηL) causes the errors to decrease. However, in the rocket engine example for $x/L = 1.0$ the errors remained at 20 percent, showing that low values of frequency or ηL will not necessarily give negligible differences between case 1 and case 2 solutions. This is a result of the difference between the assumptions used in formulating the case 1 and case 2 solutions. At low values of ηL, R values near 1.0 yield lowest errors.

To compare the two solutions accounting for thermal conductivity of the wall material, equation (21) was used to calculate the convective heat-transfer parameter with the phase lag angle obtained from the case 1 solution (eq. (B26)). The results for the rocket engines example but for an $R = 1.0$, are shown in figure 5 by the dashed lines. For the rocket engine conditions, the heat-transfer coefficients obtained from equation (21) (considering conductivity) instead of equation (15) are not significantly improved.

Differences between the case 1 and case 2 solutions do exist even if the frequency is allowed to approach zero. This has been shown in figures 5 and 6 and is expected because of the differences in the assumptions used to generate the two solutions. Generally, though it appears that a combination of a low value of ηL and R is required to minimize the differences between the heat-transfer coefficients calculated using the case 1 and case 2 solutions involving phase lag angles.

CASE 2 SOLUTION WITH MIDPOINT WALL TEMPERATURE MEASUREMENT

An attractive possibility for reducing the differences between case 1 and case 2 solutions is to measure the wall temperature at the center of the wall, $x/L = 0.5$. This will obtain a phase lag angle which should be nearly equal to the average phase lag angle. This results because the phase lag angle is approximately proportional to the distance through the wall that the wave has traveled. In figure 4(b), it can be seen that the averages of $x/L = 1.0$ and $x/L = 0$ for the same ηL values are always much closer to the case 2 base line than either surface alone is. If the wall temperature is measured at the center, the case 2 solution, in terms of phase lag angle, can possibly be used to calculate the coefficients.

The curves given in figure 5(a) for $x/L = 0.5$ show that under certain conditions the errors caused by using the case 2 solution to calculate the coefficients tend to be reduced. This is true when the comparison is made between the $x/L = 1.0$ and 0.5 case. However, a comparison of figure 5(b), $x/L = 0$, $R = 10$ to the $x/L = 0.5$ curves of figure 5(a) shows that the errors in coefficient are not reduced. Therefore, it can only be stated that, when the wall temperature is measured at the midpoint, the errors due to using the case 2 solution to calculate the coefficients tend to be reduced.

OPTIMUM PHASE LAG ANGLE

For the insulated wall case reference 1, an optimum range of phase lag angles exists. The percent error in the coefficient due to an error in measuring the phase lag angle is minimized in this range. The same results are obtained when using the case 2 solution given in this report. The equation for an incremental change in convective heat-transfer coefficient is obtained from equation (15) and is

$$\frac{1}{h}\frac{\partial h}{\partial \varphi} = -\frac{2}{\sin 2\varphi} \tag{23}$$

As shown in reference 2, the minimum value exists at $\varphi = 45^{\circ}$ (0.785 rad). Furthermore, φ can range from 20° to 80° (0.349 to 1.396 rad) before the errors in h begin to increase greatly. Hence, in using the simpler case 2 solution, a guide line is established which shows that a minimum phase lag angle of 20° (0.349 rad) and a maximum of 80° (1.396 rad) are desirable (to minimize errors in h due to inaccuracies in measuring the phase lag angle). However, it appears from inspection of figure 2 that the case 1 solution imposes an upper limit on the phase lag which is dependent on both x/L and R. The upper limit is best determined by inspection of figure 2 by examining the slope of the ψ versus φ curves. When the $\Delta\psi/\Delta\varphi$ becomes large enough so that the error in the measurement φ gives an unacceptable error in ψ a lower frequency must be used. This will give smaller phase lags and better accuracy.

In figures 5 and 6 the frequencies at which a phase lag angle of 20° is reached are indicated by arrows shown on each of the curves. These indicate the start of the optimum frequency range. Higher frequencies will increase the phase lag angle. As can be seen, these frequencies are low enough (below 3 Hz) so that they should be easily obtainable with current control systems.

SOLUTION OF PROBLEM WHEN COOLANT TEMPERATURE IS DRIVEN SINUSOIDALLY

The preceding solutions have been derived for the case in which the hot-gas temperature is driven sinusoidally. The extension of this technique to the case in which the coolant temperature is driven requires the following substitutions to be made:

$$\left.\begin{aligned} h_G &\equiv -h_c' \\ h_c &\equiv -h_G' \end{aligned}\right\} \text{case 1}$$

$$\left.\begin{aligned} h_G &\equiv h_c' \\ h_c &\equiv h_G' \end{aligned}\right\} \text{case 2}$$

$$R \equiv R_c = \frac{h_c'}{h_G'} = \frac{\overline{T}(0) - \overline{T}_c}{\overline{T}_G - \overline{T}(L)}$$

$$T_c \equiv T_G' \qquad T_G \equiv T_c' \qquad \Delta T_G \equiv \Delta T_c'$$

For the case 1 and case 2 solutions these substitutions change the stated location of the coolant-wall surface to $x = 0$ and that of the hot-gas wall surface to $x = L$ (i.e., $x \equiv 0$ at the surface on which the fluid temperature is oscillated). Making these substitutions allows the charts in figures 2 and 3 to be used to calculate the heat-transfer coefficients when the coolant temperature is varied sinusoidally instead of the hot-gas temperature. When making these substitutions in figures 2 and 3 the negative sign of the coefficient is discarded and only the absolute value of the convective heat-transfer parameter ψ is used.

SUMMARY OF RESULTS

An analytical investigation has been performed in which the temperature response of a convectively heated and cooled plate to a sinusoidally varying driving temperature is determined. The temperature response of the wall can be used to determine both convective heat-transfer coefficients. Two independent methods for calculating the coefficients are presented. One uses the phase angle of the measured wall temperature lag behind the driven (hot gas or coolant) temperature. The other method uses the ratio of the amplitude of the wall temperature to the amplitude of the driven temperature.

Each of these methods requires two independent relations between the coefficients in order to solve for the coefficients. The first relation is knowledge of the sinusoidal variation of the driven fluid and wall temperatures with time, so that the phase lag angle between these quantities (or amplitude ratio) can be determined. The second requires the time averaged values of the driven, wall, and coolant temperatures. These are used to calculate the convective heat-transfer coefficient ratio. These two relations provide the information needed to calculate the coefficients.

Two solutions for the wall temperature response are presented. The first, case 1, uses distributed wall properties and accounts for low thermal diffusivities and thick walls. In this case, the location of the measured wall temperature is important and the solutions for the convective heat-transfer coefficients in terms of phase lag angle or amplitude ratio are lengthy. Charts giving the solutions for the coefficients over a selected range of parameters are included. The other solution, case 2, assumes the wall has a lumped capacitance and thermal conductivity. In this case, the solutions for the convective heat-transfer coefficients using either phase lag angle or amplitude ratio are simple. The solutions are analyzed by comparing the convective heat-transfer coefficients calculated using the distributed solution to those calculated using the lumped solution.

Measuring the wall temperature at the midpoint in the wall tends to minimize the errors in heat-transfer coefficients when the lumped wall property solution is used to calculate them.

An optimum range of phase lag angles exists within which minimum errors in convective heat-transfer coefficients result from errors in measurement of phase lag angles.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, August 5, 1969,
122-29.

APPENDIX A

SYMBOLS

c	specific heat of wall material, Btu/(lbm)(^{o}R); J/(kg)(K)
e	natural logarithm base, 2.71828
$\overline{F}(\tau)$	part of product solution for $\theta(x, \tau)$ which is independent of x
f	frequency of temperature oscillation; Hz
h	time averaged convective heat-transfer coefficient independent of the impressed sinusoidal temperature variation, Btu/(in.^{2})(sec)(^{o}R); W/(m^2)(K)
i	imaginary number, $\sqrt{-1}$
K	thermal conductivity of wall material, Btu/(in.)(sec)(^{o}R); J/(m)(sec)(K)
L	thickness of wall material, in.; m
$\mathscr{L}[T]$	Laplace transform of temperature
M, N, P, V	functional notation
q	heat flux per unit area, Btu/(in.^{2})(sec); W/(m^2)
R	ratio of convective heat-transfer coefficients, h_c/h_G
S	Laplace transform variable
T	temperature (temperature of wall when not subscripted), ^{o}R; K
ΔT	half amplitude of temperature oscillation, ^{o}R; K
$\overline{T}$	average or mean value of temperature, ^{o}R; K
$T(L, \tau)$	wall temperature, function of time at $x = L$, ^{o}R; K
$T(0, \tau)$	wall temperature, function of time at $x = 0$, ^{o}R; K
$T(x, \tau)$	wall temperature, function of distance (measured into wall from hot surface) and time, ^{o}R; K
$T_s(x)$	steady-state part of $T(x, \tau)$, ^{o}R; K
U	modified overall heat-transfer coefficient which accounts for the thermal conductivity of the wall material, Btu/(in.^{2})(sec)(^{o}R); W/(m^2)(K)
$u(\tau)$	unit step function
$X(x)$	part of the product solution for $\theta(x, \tau)$ which is independent of time, ^{o}R; K
x	distance measured from the heated surface of the wall into wall toward cooled surface, in.; m

α thermal diffusivity, $K/\rho c$, in.^{2}/sec; m^2/sec

η frequency and wall property parameter, $\sqrt{\omega/2\alpha}$, 1/in.; 1/m

θ transient part of appropriate wall temperature solution, $^{\circ}$R; K

ρ density of wall material, lbm/in.^{3}; g/m^3

τ time, sec

φ phase lag angle between the driving gas temperature and the responding wall temperature, deg; rad

ψ convective heat-transfer parameter $K\eta/h$, dimensionless

ω angular velocity of temperature oscillation, $2\pi f$ rad/sec

Subscripts:

c coolant

cw coolant side of wall, x = L

G hot gas or fluid

GW hot-gas side of wall, x = 0

m maximum value

r revised $\theta_m/\Delta T_G$ by using first-order solution to correct the value to range from 0 to 1.0.

case 2 refers to values calculated using the appropriate case 2 solutions

0 refers to conditions before the start of the temperature oscillation

1 case 2 first-order differential equation solution not accounting for the thermal conductivity and using lumped system

2 case 2 first-order differential equation solution accounting for the thermal conductivity using modified overall heat-transfer coefficient

Superscript:

' solutions in which the coolant temperature is driven sinusoidally. In appendix B the prime indicates a derivative

APPENDIX B

DERIVATION OF THE CASE 1 SOLUTIONS - DISTRIBUTED WALL PROPERTIES

Figure 1(a) shows a wall through which heat is being transferred from one fluid to another. The heat is transferred across the fluid-wall boundaries convectively. It is required that the convective heat-transfer coefficients at the plate surfaces be determined experimentally by measuring the two fluid temperatures and only one wall temperature. This can be accomplished with the given information by oscillating one of the fluid temperatures sinusoidally and measuring either the phase lag angle between the fluid and wall temperature or the amplitudes of the fluid and wall temperature. A sketch showing what the wall temperature might look like is given in figure 1(b). The solutions for the coefficients are found by first finding the equation for the wall temperature response to the sinusoidally driven fluid temperature and then solving this equation in terms of the known quantities for the coefficients.

General Solution

The response of a convectively heated and cooled wall to a sinusoidally varying hot-gas temperature is derived here. The solution accounts for the thermal properties of the wall material being distributed throughout the wall. The following assumptions are used in the derivation:

(1) The heat flows through the plate in the x-direction only (one-dimensional heat conduction).

(2) The wall or plate properties (density, specific heat, and thermal conductivity) are constant.

(3) The convective heat-transfer coefficients on both sides of the wall are constant.

(4) The coolant temperature is constant.

The second-order partial differential equation for transient, one-dimensional, heat conduction in a plate is solved. The boundary conditions are that the plate is convectively heated on one side by a fluid having a sinusoidally varying temperature and cooled on the other side by a fluid which is not significantly affected by the oscillating heat flux from the wall. The solution follows.

The differential equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \tag{B1}$$

where $\alpha = K/\rho c$, the thermal diffusivity.

The boundary conditions are

$$h_G \left[T_G - T(0, \tau)\right] = -K \frac{\partial T(0, \tau)}{\partial x} \qquad \text{at } x = 0 \tag{B2}$$

$$h_c \left[T(L, \tau) - T_c\right] = -K \frac{\partial T(L, \tau)}{\partial x} \qquad \text{at } x = L \tag{B3}$$

The driving temperature is

$$T_G = \Delta T_G\, e^{-i\omega\tau} + \overline{T}_G \tag{B4}$$

Assume the solution has the form of a transient plus a steady-state value then

$$\theta(x, \tau) = T(x, \tau) - T_S(x) \tag{B5}$$

where T_S is the steady-state temperature distribution prior to driving $T_G(\tau)$ and $T(x,\tau)$ is the temperature response of the wall to the harmonically driven hot-gas temperature. The solution for $\theta(x, \tau)$ follows.

Using boundary condition equation (B2) and substituting for T_G the driving gas temperature, equation (B4), yields, after subtracting the steady-state heat transfer

$$\Delta T_G\, e^{-i\omega\tau} - \theta(0, \tau) = -\frac{K}{h_G} \frac{\partial \theta(0, \tau)}{\partial x} \tag{B6}$$

Boundary condition equation (B3) can be rewritten as

$$\theta(L, \tau) = -\frac{K}{h_c} \frac{\partial \theta(L, \tau)}{\partial x} \tag{B7}$$

The differential equation becomes

$$\frac{\partial^2 \theta(x, \tau)}{\partial x^2} = \frac{1}{\alpha} \frac{\partial \theta(x, \tau)}{\partial \tau} \tag{B8}$$

Assume a product solution then

$$\theta = X(x) \cdot \overline{F}(\tau) \tag{B9}$$

$$\frac{\partial \theta}{\partial x} = \overline{F}(\tau)X'(x)$$

$$\frac{\partial \theta}{\partial \tau} = X(x)\overline{F}'(\tau)$$

$$\frac{\partial^2 \theta}{\partial x^2} = \overline{F}(\tau)X''(x)$$

Substituting this into differential equation (B8) gives

$$\overline{F}(\tau)X''(x) = \frac{1}{\alpha} X(x)\overline{F}(\tau)$$

$$\frac{X''(x)}{X(x)} = \frac{1}{\alpha}\frac{\overline{F}'(\tau)}{\overline{F}(\tau)} \tag{B10}$$

Since either side of equation (B9) is independent of the variable appearing on the other side, it is concluded that either side must be equal to a constant. This constant is assumed to be λ^2 and from equation (B9) the following equations have been written:

$$X'' = \lambda^2 X \tag{B11}$$

$$\overline{F}' = \lambda^2 \alpha \overline{F} \tag{B12}$$

Three choices for λ^2 are possible; $\lambda^2 < 0$, $\lambda^2 = 0$, $\lambda^2 > 0$. If $\lambda^2 > 0$, the solution will not be periodic in x; therefore, $\lambda^2 > 0$ is rejected. If $\lambda^2 = 0$, the solution will not be periodic; therefore $\lambda^2 = 0$ is rejected. If $\lambda^2 < 0$, a solution exist and is found as follows:

From equation (B11)

$$X'' + \lambda^2 X = 0$$

The characteristic is

$$m^2 + \lambda^2 = 0$$

From which

$$m = i\lambda$$

From Wiley (ref. 5, p. 88)

$$X = c_1\, e^{i\lambda x} + c_2\, e^{-i\lambda x}$$

From the derivation for $\overline{F}$ given below in which the equation for λ^2 is deduced, the following equations for λ are derived:

$$\lambda^2 = \frac{i\omega}{\alpha}$$

$$\lambda = \sqrt{\frac{i\omega}{\alpha}}$$

$$\lambda = \pm\sqrt{i}\sqrt{\frac{\omega}{\alpha}}$$

$$\lambda = \pm\,(1 + i)\sqrt{\frac{\omega}{2\alpha}}$$

Then

$$X = c_1\, e^{\pm(1+i)\sqrt{\omega/2\alpha}\, x} + e^{\mp(1+i)\sqrt{\omega/2\alpha}\, x} \tag{B13}$$

From equation (B12)

$$\frac{\overline{F}'}{\overline{F}} = -\lambda^2\alpha$$

$$\ln \overline{F} = -\lambda^2\alpha\tau$$

$$\overline{F} = c\, e^{-\lambda^2\alpha\tau}$$

If λ^2 is real, $\overline{F}$ cannot be periodic since

$$e^{-\lambda^2 \alpha\tau} = \cosh \lambda^2 \alpha\tau - \sinh \lambda^2 \alpha\tau$$

But if λ^2 is imaginary, then

$$e^{-i\lambda^2 \alpha\tau} = \cos \lambda^2 \alpha\tau - i \sin \lambda^2 \alpha\tau$$

which is periodic as required. If $\lambda^2 = i\omega/\alpha$, where ω is the angular velocity. The $\lambda^2 \alpha\tau$ terms in this equation reduce to the usual $\omega\tau$ form. Hence

$$\overline{F} = c\ e^{-i\omega\tau} \tag{B14}$$

Substituting equations (B13) and (B14) into (B9) gives

$$\theta = c\ e^{-i\omega\tau} \left[c_1\ e^{\pm i \sqrt{(\omega/2\alpha)} x \mp \sqrt{(\omega/2\alpha)} x} + c_2\ e^{\mp i \sqrt{(\omega/2\alpha)} x \pm \sqrt{(\omega/2\alpha)} x} \right] \tag{B15}$$

The choice of signs in the exponent is arbitrary. Combining the constants ($cc_1 = A$ and $cc_2 = B$) and rewriting this equation gives

$$\theta = A\ e^{-\eta x}\ e^{-i(\omega\tau - \eta x)} + B\ e^{\eta x}\ e^{-i(\omega\tau + \eta x)} \tag{B16}$$

where $\eta \equiv \sqrt{\omega/2\alpha}$.

Application of the boundary conditions requires the $\partial\theta/\partial x$. This is given by

$$\frac{\partial\theta}{\partial x} = A\ e^{-\eta x - i(\omega\tau - \eta x)} (-\eta + i\eta) + B\ e^{\eta x - i(\omega\tau + \eta x)} (\eta - i\eta) \tag{B17}$$

$$\frac{\partial\theta}{\partial x} = \eta\ A(i - 1) e^{-\eta x - i(\omega\tau - \eta x)} + \eta\ B(1 - i) e^{\eta x - i(\omega\tau + \eta x)} \tag{B18}$$

Now equation (B6) gives

$$\Delta T_G e^{-i\omega\tau} - \theta(0,\tau) = \frac{-K}{h_G} \frac{\partial\theta(0,\tau)}{\partial x}$$

$$\Delta T_G e^{-i\omega\tau} - A e^{-i\omega\tau} - B e^{-i\omega\tau} = -\frac{K\eta}{h_G}(A - B)(i - 1)e^{-i\omega\tau}$$

$$\Delta T_G - A - B = \frac{K\eta}{h_G}(B - A)(i - 1)$$

$$\Delta T_G = B\left[\frac{K\eta}{h_G}(i - 1) + 1\right] - A\left[\frac{K\eta}{h_G}(i - 1) - 1\right] \tag{B19}$$

Applying the boundary condition equation (B7) at $x = L$ gives

$$\theta(L,\tau) = -\frac{K}{h_c} \frac{\partial\theta(L,\tau)}{\partial x}$$

$$A e^{-\eta L} e^{-i(\omega\tau-\eta L)} + B e^{\eta L} e^{-i(\omega\tau+\eta L)} = -\frac{K\eta}{h_c} A(i - 1)e^{-\eta L-i(\omega\tau-\eta L)} - \frac{K\eta}{h_c} B(1 - i)e^{\eta L-i(\omega\tau+\eta L)}$$

$$A\left[1 + \frac{K\eta}{h_c}(i - 1)\right] e^{-\eta L-i(\omega\tau-\eta L)} = -B\left[1 + \frac{K\eta}{h_c}(1 - i)\right] e^{\eta L-i(\omega\tau+\eta L)}$$

$$\frac{A}{B} = \frac{-\left[1 + \frac{K\eta}{h_c}(1 - i)\right]}{\left[1 + \frac{K\eta}{h_c}(i - 1)\right]} e^{2\eta L-i2\eta L} \tag{B20}$$

Using this expression and substituting in equation (B19), which was obtained from boundary condition equation (B6), the equations for the constants A and B have been obtained as follows:

$$\Delta T_G = B\left[\frac{K\eta}{h_G}(i - 1) + 1\right] + B\frac{\left[1 + \frac{K\eta}{h_c}(1 - i)\right]}{\left[1 + \frac{K\eta}{h_c}(i - 1)\right]}\left[\frac{K\eta}{h_G}(i - 1) - 1\right]e^{2\eta L - i2\eta L}$$

$$B = \frac{\Delta T_G\left[1 + \frac{K\eta}{h_c}(i - 1)\right]}{\left[\frac{K\eta}{h_G}(i - 1) + 1\right]\left[1 + \frac{K\eta}{h_c}(i - 1)\right] + \left[1 + \frac{K\eta}{h_c}(1 - i)\right]\left[\frac{K\eta}{h_G}(i - 1) - 1\right]e^{2\eta L - i2\eta L}} \tag{B21}$$

$$A = \frac{-\Delta T_G\left[1 + \frac{K\eta}{h_c}(1 - i)\right]e^{2\eta L - i2\eta L}}{\left[\frac{K\eta}{h_G}(i - 1) + 1\right]\left[1 + \frac{K\eta}{h_c}(i - 1)\right] + \left[1 + \frac{K\eta}{h_c}(1 - i)\right]\left[\frac{K\eta}{h_G}(i - 1) - 1\right]e^{2\eta L - i2\eta L}} \tag{B22}$$

Working with the common denominator in the equations for the constants A and B (eqs. (B21) and (B22)) and defining $\psi \equiv K\eta/h$ results in

$$\left[\psi_G(i - 1) + 1\right]\left[\psi_c(i - 1) + 1\right] + \left[-\psi_c(i - 1) + 1\right]\left[\psi_G(i - 1) - 1\right]e^{2\eta L - i2\eta L}$$

$$\left[(1 - \psi_G) + i\psi_G\right]\left[(1 - \psi_c) + i\psi_c\right] + \left[(1 + \psi_c) - i\psi_c\right]\left[-(1 + \psi_G) + i\psi_G\right]e^{2\eta L - i2\eta L}$$

$$1 - \psi_G - \psi_c + \psi_G\psi_c - \psi_G\psi_c + i(\psi_G - \psi_G\psi_c + \psi_c - \psi_G\psi_c)$$

$$+ \left[-1 - \psi_c - \psi_G - \psi_G\psi_c + \psi_G\psi_c + i(\psi_c + \psi_G\psi_c + \psi_G + \psi_G\psi_c)\right]e^{2\eta L - i2\eta L}$$

$$1 - (\psi_G + \psi_c) + i(\psi_G + \psi_c - 2\psi_G\psi_c)$$

$$+ e^{2\eta L}\left[-(1 + \psi_c + \psi_G) + i(\psi_G + \psi_c + 2\psi_G\psi_c)\right](\cos 2\eta L - i \sin 2\eta L)$$

$$1 - (\psi_G + \psi_c) + e^{2\eta L}\left[-(1 + \psi_c + \psi_G)\cos 2\eta L + (\psi_G + \psi_c + 2\psi_G\psi_c)\sin 2\eta L\right]$$

$$+ i\left[(\psi_G + \psi_c - 2\psi_G\psi_c) + (\psi_G + \psi_c + 2\psi_G\psi_c)\cos 2\eta L\, e^{2\eta L} + (1 + \psi_c + \psi_G)\sin 2\eta L\, e^{2\eta L}\right]$$

For simplicity this expression is rewritten as a + ib. Then

$$A = \frac{-\Delta T_G\left[(1 + \psi_c) - i\psi_c\right]e^{2\eta L - 2\eta L}}{a + ib}$$

$$= \frac{-\Delta T_G\left[(1 + \psi_c) - i\psi_c\right](\cos 2\eta L - i \sin 2\eta L)e^{2\eta L}}{a + ib}$$

$$= \frac{-\Delta T_G\, e^{2\eta L}\left\{\left[(1 + \psi_c)\cos 2\eta L - \psi_c \sin 2\eta L\right] - i\left[\psi_c \cos 2\eta L + (1 + \psi_c)\sin 2\eta L\right]\right\}(a - ib)}{a^2 + b^2}$$

Rewriting the preceeding equation in simpler form using constants c and d as defined by equations (B30) and (B31), respectively, gives

$$A = \frac{-\Delta T_G\, e^{2\eta L}(c - id)(a - ib)}{a^2 + b^2} = \frac{-\Delta T_G\, e^{2\eta L}\left[ca - db - i(da + cb)\right]}{a^2 + b^2} \tag{B23}$$

$$B = \frac{\Delta T_G\left[(1 - \varphi_c) + i\varphi_c\right]}{(a + ib)} = \frac{\Delta T_G(e + if)(a - ib)}{a^2 + b^2} = \frac{\Delta T_G\left[ea + fb + i(fa - eb)\right]}{a^2 + b^2} \tag{B24}$$

where e and f are defined in equations (B32) and (B33). The solution for θ, making use of equations (B23) and (B24), is as follows:

$$\theta = \frac{-\Delta T_G\, e^{2\eta L - \eta x}\left[ca - db - i(da + cb)\right]e^{i\eta x} + \Delta T_G\, e^{\eta x}\left[ea + fb + i(fa - eb)\right]e^{-i\eta x}}{a^2 + b^2}\, e^{-i\omega\tau}$$

$$\theta = \frac{-\Delta T_G\, e^{\eta(2L - x)}\left[ca - db - i(da + cb)\right](\cos \eta x + i \sin \eta x) + \Delta T_G\, e^{\eta x}\left[ea + fb + i(fa - eb)\right](\cos \eta x - i \sin \eta x)}{a^2 + b^2}\, e^{-i\omega\tau}$$

$$\theta = \frac{+\Delta T_G}{a^2 + b^2}\left(-e^{\eta(2L-x)}\left[(ca - db)\cos \eta x + (da + cb)\sin \eta x\right] + e^{\eta x}\left[(ea + fb)\cos \eta x + (fa - eb)\sin \eta x\right]\right.$$
$$\left. + i\left\{\left[-(ca - db)\sin \eta x + (da + cb)\cos \eta x\right]e^{\eta(2L-x)} + \left[(fa - eb)\cos \eta x - (ea + fb)\sin \eta x\right]e^{\eta x}\right\}\right) e^{-i\omega\tau} \tag{B25}$$

The phase lag between the wall and the driving hot gas is defined as φ and is

$$\varphi = \tan^{-1} \frac{\left[-(ca - db)\sin \eta x + (da + cb)\cos \eta x\right]e^{\eta(2L-x)} + \left[(fa - eb)\cos \eta x - (ea + fb)\sin \eta x\right]e^{\eta x}}{\left[(ea + fb)\cos \eta x + (fa - eb)\sin \eta x\right]e^{\eta x} - \left[(ca - db)\cos \eta x + (da + cb)\sin \eta x\right]e^{\eta(2L-x)}} \tag{B26}$$

The amplitude ratio is

$$\frac{\theta_m}{\Delta T_G} = \frac{1}{a^2+b^2}\left(\left\{\left[-(ca - db)\sin \eta x + (da + cb)\cos \eta x\right]e^{\eta(2L-x)} + \left[(fa - eb)\cos \eta x - (ea+fb)\sin \eta x\right]e^{\eta x}\right\}^2\right.$$
$$\left. + \left\{\left[(ea + fb)\cos \eta x + (fa - eb)\sin \eta x\right]e^{\eta x} - \left[(ca - db)\cos \eta x + (da + cb)\sin \eta x\right]e^{\eta(2L-x)}\right\}^2\right)^{1/2} \tag{B27}$$

where

$$a = 1 - (\psi_G + \psi_c) + e^{2\eta L}\left[(\psi_G + \psi_c + 2\psi_G\psi_c)\sin 2\eta L - (1 + \psi_c + \psi_G)\cos 2\eta L\right] \tag{B28}$$

$$b = (\psi_G + \psi_c - 2\psi_G\psi_c) + e^{2\eta L}\left[(\psi_G + \psi_c + 2\psi_G\psi_c)\cos 2\eta L + (1 + \psi_c + \psi_G)\sin 2\eta L\right] \tag{B29}$$

$$c = (1 + \psi_c)\cos 2\eta L - \psi_c \sin 2\eta L \tag{B30}$$

$$d = \psi_c \cos 2\eta L + (1 + \psi_c)\sin 2\eta L \tag{B31}$$

$$e = 1 - \psi_c \tag{B32}$$

$$f = \psi_c \tag{B33}$$

The important equations written in nondimensional form are as follows:

General solution for wall temperature

$$\frac{\theta}{\Delta T_G} = \frac{1}{a^2+b^2}\left(\left[(ea+fb)\cos\eta L\frac{x}{L}+(fa-eb)\sin\eta L\frac{x}{L}\right]e^{\eta L(x/L)}\right.$$

$$-\left[(ca-db)\cos\eta L\frac{x}{L}+(da+cb)\sin\eta L\frac{x}{L}\right]e^{\eta L(2-x/L)}$$

$$+i\left\{\left[-(ca-db)\sin\eta L\frac{x}{L}+(da+cb)\cos\eta L\frac{x}{L}\right]e^{\eta L[2-(x/L)]}\right.$$

$$\left.\left.+\left[(fa-eb)\cos\eta L\frac{x}{L}-(ea+fb)\sin\eta L\frac{x}{L}\right]e^{\eta L(x/L)}\right\}\right)e^{-i\omega\tau} \tag{B34}$$

or

$$\frac{\theta}{\Delta T_G} = \frac{\theta_m}{\Delta T_G}\,e^{-i(\omega\tau-\varphi)} \tag{B35}$$

where

$$\varphi = \tan^{-1}\frac{\left[-(ca-db)\sin\eta L\frac{x}{L}+(da+cb)\cos\eta L\frac{x}{L}\right]e^{\eta L[2-(x/L)]}+\left[(fa-eb)\cos\eta L\frac{x}{L}-(ea+fb)\sin\eta L\frac{x}{L}\right]e^{\eta L(x/L)}}{\left[(ea+fb)\cos\eta L\frac{x}{L}+(fa-eb)\sin\eta L\frac{x}{L}\right]e^{\eta L(x/L)}-\left[(ca-db)\cos\eta L\frac{x}{L}+(da+cb)\sin\eta L\frac{x}{L}\right]e^{\eta L[2-(x/L)]}} \tag{B36}$$

$$\frac{\theta_m}{\Delta T_G} = \frac{1}{a^2+b^2}\left(\left\{-\left[(ca-db)\sin\eta L\frac{x}{L}-(da+cb)\cos\eta L\frac{x}{L}\right]e^{\eta L[2-(x/L)]}\right.\right.$$

$$\left.+\left[(fa-eb)\cos\eta L\frac{x}{L}-(ea+fb)\sin\eta L\frac{x}{L}\right]e^{\eta L(x/L)}\right\}^2$$

$$+\left\{\left[(ea+fb)\cos\eta L\frac{x}{L}+(fa-eb)\sin\eta L\frac{x}{L}\right]e^{\eta L(x/L)}\right.$$

$$\left.\left.-\left[(ca-db)\cos\eta L\frac{x}{L}+(da+cb)\sin\eta L\frac{x}{L}\right]e^{\eta L[2-(x/L)]}\right\}^2\right)^{1/2} \tag{B37}$$

Derivation of Convective Heat-Transfer Parameters as Function of Forcing Frequency, Wall Material Properties, and Phase Lag

The equation which gives the convective heat-transfer parameters, ψ_G and ψ_c, in terms of the forcing frequency of the hot-gas temperature, wall material properties (including thermal conductivity), and phase lag angle is derived in this section.

Equation (B26) is rewritten as

$$\tan \varphi\,(ea + fb)\cos \eta x\, e^{\eta x} + \tan \varphi(fa - eb)\sin \eta x\, e^{\eta x} - \tan \varphi(ca - db)\cos \eta x\, e^{\eta(2L-x)}$$

$$- \tan \varphi(da + cb)\sin \eta x\, e^{\eta(2L-x)} + (ea + fb)\sin \eta x\, e^{\eta x} - (fa - eb)\cos \eta x\, e^{\eta x}$$

$$+ (ca - db)\sin \eta x\, e^{\eta(2L-x)} - (da + cb)\cos \eta x\, e^{\eta(2L-x)} = 0$$

$$(ea + fb)(\tan \varphi \cos \eta x + \sin \eta x)e^{\eta x} + (fa - eb)(\tan \varphi \sin \eta x - \cos \eta x)e^{\eta x}$$

$$- (ca - db)(\tan \varphi \cos \eta x - \sin \eta x)e^{\eta(2L-x)} - (da + cb)(\tan \varphi \sin \eta x + \cos \eta x)e^{\eta(2L-x)} = 0$$

$$(ea + fb)(\sin \varphi \cos \eta x + \cos \varphi \sin \eta x)e^{\eta x} + (fa - eb)(\sin \varphi \sin \eta x - \cos \varphi \cos \eta x)e^{\eta x}$$

$$- (ca - db)(\sin \varphi \cos \eta x - \cos \varphi \sin \eta x)e^{\eta(2L-x)}$$

$$- (da + cb)(\sin \varphi \sin \eta x + \cos \varphi \cos \eta x)e^{\eta(2L-x)} = 0$$

$$(ea + fb)\sin(\varphi + \eta x) - (fa - eb)\cos(\varphi + \eta x) - e^{2\eta(L-x)}\Big[(ca - db)\sin(\varphi - \eta x)$$

$$+ (da + cb)\cos(\varphi - \eta x)\Big] = 0 \qquad \text{(B38)}$$

Using equation (B38), factoring out the terms having φ_c and φ_G in them, and defining the coefficients as

$$\underline{A} \equiv -\sin(\varphi + \eta x) + e^{2\eta L} \cos(\varphi + \eta x - 2\eta L) + e^{2\eta(2L-x)} \sin(\varphi - \eta x)$$

$$-e^{2\eta(L-x)} \cos(\varphi - \eta x + 2\eta L) \qquad \text{(B39)}$$

$$\underline{B} \equiv \cos(\varphi + \eta x) - e^{2\eta L} \sin(\varphi + \eta x - 2\eta L) - e^{2\eta(2L-x)} \cos(\varphi - \eta x)$$

$$+ e^{2\eta(L-x)} \sin(\varphi - \eta x + 2\eta L) \qquad \text{(B40)}$$

$$\underline{C} \equiv \sin(\varphi + \eta x) + e^{2\eta L} \sin(\varphi + \eta x - 2\eta L) + e^{2\eta(2L-x)} \sin(\varphi - \eta x)$$

$$+ e^{2\eta(L-x)} \sin(\varphi - \eta x + 2\eta L) \qquad \text{(B41)}$$

$$D \equiv -\cos(\varphi + \eta x) + e^{2\eta L} \cos(\varphi + \eta x - 2\eta L) - e^{2\eta(2L-x)} \cos(\varphi - \eta x)$$

$$+ e^{2\eta(L-x)} \cos(\varphi - \eta x + 2\eta L) \qquad \text{(B42)}$$

$$E \equiv \sin(\varphi + \eta x) - e^{2\eta L} \sin(\varphi + \eta x - 2\eta L) + e^{2\eta(2L-x)} \sin(\varphi - \eta x)$$

$$- e^{2\eta(L-x)} \sin(\varphi - \eta x + 2\eta L) \qquad \text{(B43)}$$

$$F \equiv \cos(\varphi + \eta x) + e^{2\eta L} \sin(\varphi + \eta x - 2\eta L) - e^{2\eta(2L-x)} \cos(\varphi - \eta x)$$

$$- e^{2\eta(L-x)} \sin(\varphi - \eta x + 2\eta L) \qquad \text{(B44)}$$

$$G \equiv -\sin(\varphi + \eta x) - e^{2\eta L} \cos(\varphi + \eta x - 2\eta L) + e^{2\eta(2L-x)} \sin(\varphi - \eta x)$$

$$+ e^{2\eta(L-x)} \cos(\varphi - \eta x + 2\eta L) \qquad \text{(B45)}$$

the desired equation is

$$2(F + G)\psi_G\psi_c^2 + 2\underline{C}\psi_c^2 + 2(\underline{C} + D)\psi_G\psi_c + 2\underline{A}\psi_c + (\underline{A} + \underline{B})\psi_G + E = 0 \qquad \text{(B46)}$$

If ψ_G is a known quantity, the equation reduces to a quadratic which is easily solved. If ψ_c is known, the equation is linear and again can be solved easily. However, if neither ψ_G or ψ_c are known, a relation must be established between them to solve the equation. This relation exists in the steady-state solution for heat transfer across a plate.

$$h_G\left[T_G - T_w(0)\right] = h_c\left[T_w(L) - T_c\right] \qquad \text{(B47)}$$

Rearranging equation (B47) gives

$$\frac{h_c}{h_G} = \frac{T_G - T(0)}{T(L) - T_c} \tag{B48}$$

Defining the ratio in equation (B48) as R results in

$$R \equiv \frac{h_c}{h_G} = \frac{T_G - T(0)}{T(L) - T_c} \tag{B49}$$

since $\psi = K\eta/h$

$$R = \frac{\psi_G}{\psi_c} \tag{B50}$$

Substituting equation (B50) into equation (B46) gives

$$\psi_c^3 + \frac{\left[\underline{C} + R(\underline{C} + D)\right]}{R(F + G)}\psi_c^2 + \frac{2\underline{A} + R(\underline{A} + \underline{B})}{2R(F + G)}\psi_c + \frac{E}{2R(F + G)} = 0 \tag{B51}$$

The solution of this cubic equation can be obtained using the classical approach. The positive, nonzero, root will yield the desired convective heat-transfer parameter ψ_c. From equation (B50), then

$$\psi_G = R\psi_c \tag{B52}$$

APPENDIX C

DERIVATION OF THE CASE 2 SOLUTIONS - LUMPED WALL PROPERTIES

The basic problem is the same as has been stated in appendix B. However, appendix B gives the solution for the wall temperature obtained using the distributed wall property differential equation. In this derivation it is assumed either that the wall properties (density and specific heat) can be lumped and that the temperature gradient across the wall is negligible or that the temperature gradient across the wall (effect of finite thermal conductivity) can be accounted for by using a modified overall heat-transfer coefficient and lumping the wall properties at the point where the wall temperature is measured.

The solution for the temperature response of the wall to a sinusoidally varying fluid temperature is derived in the first section of this appendix assuming no wall temperature gradient to exist. This solution includes the exponential decaying terms (starting transient), an offset of the average temperatures due to a change of the average fluid temperatures, and the steady-state sinusoidal oscillations. The starting transient is of interest in determining the time required to reach steady-state oscillating conditions.

The steady-state oscillation portion of the solution has been rearranged to give the heat-transfer coefficients as functions of, among other things, either the steady-state phase lag angle or the amplitude ratio. These equations are presented in the text.

The second section of this appendix presents the solution for the steady-state oscillating wall temperature arrived at in the first section but modified to account for the thermal conductivity. This is accomplished by defining an overall heat-transfer coefficient to exist between the appropriate fluid and the point in the wall where the wall temperature is measured. The density and the specific heat are assumed to be concentrated at the point in the wall where the temperature is measured. The resulting solution for the wall temperature is then rearranged to give the convective heat-transfer parameter in terms of, among other parameters, either the phase lag angle or the amplitude ratio.

Response of a Convectively Heated and Cooled Wall to a Sinusoidally Varying Hot-Gas Temperature Neglecting the Effect of Thermal Conductivity

The hot-gas temperature varies sinusoidally and can be written as

$$T_G = \overline{T}_G + \Delta T_G \sin \omega \tau \tag{C1}$$

The heat balance across the wall is formulated as

$$\rho c L \frac{dT}{d\tau} = h_G(T_G - T) - h_c(T - T_c) \tag{C2}$$

Substituting equation (C1) for T_G in equation (C2) and dividing through by $\rho c L$ gives

$$\frac{dT}{d\tau} = \frac{h_G}{\rho c L}(\overline{T}_G + \Delta T_G \sin \omega\tau - T) - \frac{h_c}{\rho c L}(T - T_c) \tag{C3}$$

The solution of this first-order differential equation has been accomplished using Laplace transform techniques. Hence the solution yields information about the time required for the wall to reach a steady-state oscillating condition after the start of the hot-gas temperature oscillation.

Making the Laplace transformation in equation (C3) yields

$$S\mathscr{L}[T] - T_0 = \frac{h_G\overline{T}_G}{\rho c L S} + \frac{h_G \Delta T_G}{\rho c L}\,\frac{\omega}{S^2 + \omega^2} - \frac{h_G}{\rho c L}\mathscr{L}[T] - \frac{h_c}{\rho c L}\mathscr{L}[T] + \frac{h_c T_c}{\rho c L S} \tag{C4}$$

Solving for $\mathscr{L}[T]$ in equation (C4) gives

$$\mathscr{L}[T] = \frac{T_0 + \dfrac{h_G\overline{T}_G + h_c T_c}{\rho c L S} + \dfrac{h_G \Delta T_G \omega}{\rho c L(S^2 + \omega^2)}}{S + \dfrac{h_G + h_c}{\rho c L}} \tag{C5}$$

Rearranging equation (C5) by dividing both the numerator and denominator by $\rho c L$ gives

$$\mathscr{L}[T] = \frac{T_0}{\left(S + \dfrac{h_G}{\rho c L} + \dfrac{h_c}{\rho c L}\right)} + \frac{\dfrac{h_G}{\rho c L}\overline{T}_G + \dfrac{h_c}{\rho c L} T_c}{S\left(S + \dfrac{h_G}{\rho c L} + \dfrac{h_c}{\rho c L}\right)} + \frac{\dfrac{h_G}{\rho c L}\Delta T_G \omega}{\left(S + \dfrac{h_G}{\rho c L} + \dfrac{h_c}{\rho c L}\right)(S^2 + \omega^2)} \tag{C6}$$

The last two terms in equation (C6) have been rewritten using partial fractions so that the inverse transforms can be found. The resulting rewritten form of equation (C6) is

$$\mathscr{L}[T]=\frac{T_0}{\left(S+\frac{h_G+h_c}{\rho cL}\right)}+\frac{h_G\overline{T}_G+h_cT_c}{h_G+h_c}\left(\frac{1}{S}-\frac{1}{S+\frac{h_G+h_c}{\rho cL}}\right)+\frac{h_G\Delta T_G}{\rho cL}$$

$$\times\left\{\frac{\omega}{\left[\left(\frac{h_G+h_c}{\rho cL}\right)^2+\omega^2\right]\left(S+\frac{h_G+h_c}{\rho cL}\right)}-\frac{1}{\left(\frac{h_G+h_c}{\rho cL}-i\omega\right)2i(S+i\omega)}+\frac{1}{\left(\frac{h_G+h_c}{\rho cL}+i\omega\right)2i(S-i\omega)}\right\} \tag{C7}$$

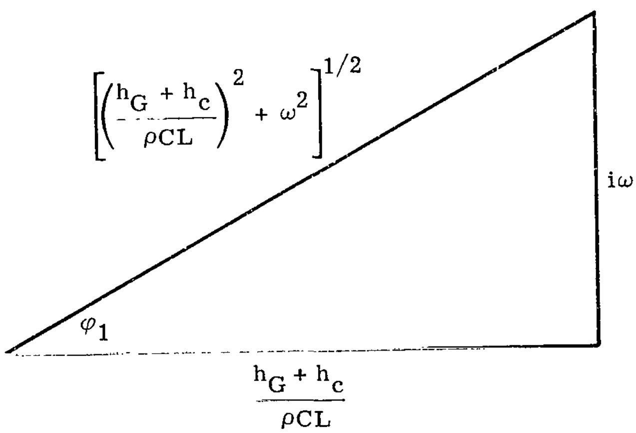
The inverse Laplace transform of equation (C7) yields the wall temperature

$$T=T_0\,e^{-[(h_G+h_c)/(\rho cL)]\tau}+\frac{h_G\overline{T}_G+h_cT_c}{h_G+h_c}\left[u(\tau)-e^{-[(h_G+h_c)/\rho cL)]\tau}\right]$$

$$+\frac{h_G\Delta T_G}{\rho cL}\left[\frac{\omega}{\left(\frac{h_G+h_c}{\rho cL}\right)^2+\omega^2}e^{-[(h_G+h_c)/(\rho cL)]\tau}-\frac{e^{-i\omega\tau}}{\left(\frac{h_G+h_c}{\rho cL}-i\omega\right)2i}+\frac{e^{i\omega\tau}}{\left(\frac{h_G+h_c}{\rho cL}+i\omega\right)2i}\right] \tag{C8}$$

The last two terms in the bracketed quantity in equation (C8) have been combined by re-writing the numerators in complex polar form and converting the resulting polar form to the equivalent trigonometric form as follows.

Define an angle φ as shown in the following sketch:



The last two terms of equation (C8) are then written as

$$\frac{h_G \Delta T_G}{\rho c L}\left[-\frac{e^{-i\omega\tau}}{\left(\frac{h_G + h_c}{\rho c L} - i\omega\right)2i} + \frac{e^{-i\omega\tau}}{\left(\frac{h_G + h_c}{\rho c L} + i\omega\right)2i}\right] = \frac{\frac{h_G \Delta T_G}{\rho c L}}{\sqrt{\left(\frac{h_G + h_c}{\rho c L}\right)^2 + \omega^2}}\left[\frac{-e^{-i(\omega\tau - \varphi)} + e^{i(\omega\tau-\varphi)}}{2i}\right]$$

$$= \frac{\frac{h_G \Delta T_G}{\rho c L}}{\sqrt{\left(\frac{h_G + h_c}{\rho c L}\right)^2 + \omega^2}} \sin(\omega\tau - \varphi)$$

Substituting this result into equation (C8) and collecting terms gives the equation for the wall temperature response to a sinusoidally driven hot-gas temperature with the initial wall temperature being T_0. The response, assuming the wall material has a large thermal conductivity, is

$$T = \overbrace{\left\{T_0 - \frac{h_G \overline{T}_G + h_c T_c}{h_G + h_c} + \frac{h_G \Delta T_G}{\rho c L} \frac{\omega}{\left[\left(\frac{h_G + h_c}{\rho c L}\right)^2 + \omega^2\right]}\right\} e^{-[(h_G + h_c)/(\rho c L)]\tau}}^{\text{Starting transient}} + \overbrace{\frac{h_G \overline{T}_G + h_c T_c}{h_G + h_c} u(\tau)}^{\text{Offset from conditions at } \tau = 0}$$

$$+ \overbrace{\frac{\frac{h_G \Delta T_G}{\rho c L}}{\sqrt{\left(\frac{h_G + h_c}{\rho c L}\right)^2 + \omega^2}} \sin(\omega\tau - \varphi)}^{\text{Steady-state oscillation}} \qquad \text{(C9)}$$

where

$$\varphi_1 \equiv \tan^{-1} \frac{\rho c L \omega}{h_G + h_c} \tag{C10}$$

$$u(\tau) = 0 \qquad \text{for} \quad \tau < 0$$

$$u(\tau) = 1 \qquad \text{for} \quad \tau > 0$$

and the amplitude ratio at steady-state oscillation is given by

$$\frac{\theta_{m1}}{\Delta T_G} = \frac{\dfrac{h_G}{\rho c L}}{\sqrt{\left(\dfrac{h_G + h_c}{\rho c L}\right)^2 + \omega^2}} \tag{C11}$$

Thermal Conductivity Accounted for in Lumped Solution

It is possible to account for the thermal conductivity in the lumped solution (eq. (C9)) by assuming that the convective heat-transfer coefficient h can be replaced by a modified overall coefficient:

$$U_G = \frac{h_G K}{K + x_G h_G} \tag{C12}$$

$$U_c = \frac{h_c K}{K + x_c h_c} \tag{C13}$$

where x_G is the distance into the wall from the hot-gas side and x_c is the distance into the wall from the coolant side. Then

$$x_c = L - x_G \tag{C14}$$

The last term in equation (C9), the steady-state oscillation portion of the solution, has been rewritten as follows:

$$\theta_1 = \frac{\dfrac{h_G \Delta T_G}{h_G + h_c}}{\sqrt{1 + \left(\dfrac{\rho c L \omega}{h_G + h_c}\right)^2}} \sin(\omega\tau - \varphi_1)$$

Using equation (C10) this expression becomes

$$\frac{\theta_1}{\Delta T_G} = \frac{1}{1 + \dfrac{h_c}{h_G}} \frac{1}{\sqrt{1 + \tan^2 \varphi_1}} \sin(\omega\tau - \varphi_1)$$

$$\frac{\theta_1}{\Delta T_G} = \frac{1}{1 + \dfrac{h_c}{h_G}} \cos \varphi_1 \sin(\omega\tau - \varphi_1) \tag{C15}$$

Substituting the modified overall coefficient U (eqs. (C12) and (C13)) for h_G and h_c, respectively, in equation (C15) yields

$$\frac{\theta_2}{\Delta T_G} = \frac{\cos \varphi_2}{1 + \dfrac{h_c(K + x_G h_G)}{h_G\left[K + (L - x_G)h_c\right]}} \sin(\omega\tau - \varphi_2)$$

$$\frac{\theta_2}{\Delta T_G} = \frac{\cos \varphi_2}{1 + \dfrac{\dfrac{K}{Lh_G} + \dfrac{x_G}{L}}{\dfrac{K}{Lh_c} + \left(1 - \dfrac{x_G}{L}\right)}} \sin(\omega\tau - \varphi_2)$$

where subscript 2 indicates that the modified overall coefficient has been used.

This equation has been rewritten to include ψ as follows: Since

$$\frac{\psi}{\eta L} = \frac{K}{hL} \tag{C16}$$

and

$$\psi_c \equiv \frac{\psi_G}{R} \tag{C17}$$

$$\frac{\theta_2}{\Delta T_G} = \frac{\cos \varphi_2}{1 + \dfrac{R\left(1 + \dfrac{x_G \eta L}{L\psi_G}\right)}{1 + \left(1 - \dfrac{x_G}{L}\right)\dfrac{\eta L R}{\psi_G}}} \sin(\omega\tau - \varphi_2) \tag{C18}$$

This equation gives the wall temperature amplitude ratio derived from the first-order differential equation and accounts for the thermal conductivity of the wall material by using a lumped system. A lumped system in this case means that the wall is divided into parts and that the conductance of each part is added to that coefficient which applies to the boundary shared by the appropriate coefficient and the assigned portion of the wall.

The phase lag (eq. (C10)) has also been reworked to account for the thermal conductivity using the lumped system approach.

Substituting equations (C12) and (C13) for h_G and h_c, respectively, in equation (C10) and using the subscript 2 to indicate the resulting phase lag angle yields

$$\varphi_2 = \tan^{-1} \frac{\rho c L \omega}{\dfrac{h_G K}{K + x_G h_G} + \dfrac{h_c K}{K + (L - x_G)h_c}} \tag{C19}$$

Multiplying both the numerator and the denominator by 2 and rearranging this equation gives

$$\varphi_2 = \tan^{-1} \frac{2}{\dfrac{2h_G}{\rho c L \omega \left(1 + \dfrac{x_G h_G L}{LK}\right)} + \dfrac{2h_c}{\rho c L \omega \left[1 + \left(1 - \dfrac{x_G}{L}\right)\dfrac{h_c L}{K}\right]}} \tag{C20}$$

Using equations (C16) and (C17) and since

$$\psi \eta L = \frac{\rho c L \omega}{2h} \tag{C21}$$

equation (C20) has been rewritten as follows:

$$\varphi_2 = \tan^{-1} \frac{2}{\dfrac{1}{\psi_G \eta L \left(1 + \dfrac{x_G}{L}\dfrac{\eta L}{\psi_G}\right)} + \dfrac{R}{\psi_G \eta L \left[1 + \left(1 - \dfrac{x_G}{L}\right)\dfrac{\eta L R}{\psi_G}\right]}} \tag{C22}$$

$$\varphi_2 = \tan^{-1} \frac{2\eta L \psi_G \left(1 + \dfrac{x_G}{L}\dfrac{\eta L}{\psi_G}\right)}{1 + \dfrac{R\left(1 + \dfrac{x_G}{L}\dfrac{\eta L}{\psi_G}\right)}{1 + \left(1 - \dfrac{x_G}{L}\right)\dfrac{\eta L R}{\psi_G}}} \tag{C23}$$

Equation (C23) gives the phase lag angle derived from the first-order differential equation and accounts for the thermal conductivity of the wall material by using a lumped system. Reworking equation (C23) yields

$$\tan \varphi_2 \left[1 + \left(1 - \frac{x_G}{L}\right)\frac{\eta L R}{\psi_G} + R\left(1 + \frac{x_G}{L}\frac{\eta L}{\psi_G}\right)\right] = 2\eta L \psi_G \left(1 + \frac{x_G}{L}\frac{\eta L}{\psi_G}\right)\left[1 + \left(1 - \frac{x_G}{L}\right)\frac{\eta L R}{\psi_G}\right]$$

$$\tan \varphi_2 \left[\psi_G + \left(1 - \frac{x_G}{L}\right)\eta L R + R\psi_G + \frac{x_G}{L}\eta L R\right] = 2\eta L \left(\psi_G + \frac{x_G}{L}\eta L\right)\left[\psi_G + \left(1 - \frac{x_G}{L}\right)\eta L R\right]$$

$$2\eta L \psi_G^2 + \left[2\frac{x_G}{L}\overline{\eta L}^2 + 2\left(1 - \frac{x_G}{L}\right)\overline{\eta L}^2 R - \tan \varphi_2 (1 + R)\right]\psi_G + \frac{2x_G}{L}\left(1 - \frac{x_G}{L}\right)\overline{\eta L}^3 R - \eta L R \tan \varphi_2 = 0$$

$$\psi_G^2 + \left\{\eta L \left[\frac{x_G}{L} + \left(1 - \frac{x_G}{L}\right)R\right] - \tan \varphi_2 \frac{(1 + R)}{2\eta L}\right\}\psi_G + \frac{x_G}{L}\left(1 - \frac{x_G}{L}\right)\overline{\eta L}^2 R - \frac{R}{2}\tan \varphi_2 = 0 \tag{C24}$$

This equation relates the convective heat-transfer parameter ψ_G to ηL, R, x/L and φ. Since $\psi_G = R\psi_c$, equation (C24) can be rewritten in terms of ψ_c as

$$\psi_c^2 + \left\{\eta L\left[\frac{x_G}{L} + \left(1 - \frac{x_G}{L}\right)R\right] - \tan\varphi_2 \frac{(1+R)}{2\eta L}\right\}\frac{\psi_c}{R} + \frac{x_G}{L}\left(1 - \frac{x_G}{L}\right)\frac{\overline{\eta L}^2}{R} - \frac{\tan\varphi_2}{2R} = 0 \quad \text{(C25)}$$

From equation (C24)

$$\psi_G = -\frac{\eta L}{2}\left[\frac{x_G}{L} + \left(1 - \frac{x_G}{L}\right)R\right] + \frac{\tan\varphi_2(1+R)}{2 \cdot 2\eta L} \pm \frac{1}{2}\left(\left\{\eta L\left[\frac{x_G}{L} + \left(1 - \frac{x_G}{L}\right)R\right] - \frac{\tan\varphi_2(1+R)}{2\eta L}\right\}^2 - 4\left[\frac{x_G}{L}\left(1 - \frac{x_G}{L}\right)(\eta L)^2 R - \frac{R}{2}\tan\varphi_2\right]\right)^{1/2} \quad \text{(C26)}$$

if $x_G/L = 0$

$$\psi_G = -\frac{\eta LR}{2} + \frac{\tan\varphi_2(1+R)}{4\eta L} \pm \frac{1}{2}\left\{\left[\eta LR - \frac{\tan\varphi_2(1+R)}{2\eta L}\right]^2 + 2R\tan\varphi_2\right\}^{1/2}$$

if $x_G/L = 1$

$$\psi_G = -\frac{\eta L}{2} + \frac{\tan\varphi_2(1+R)}{4\eta L} \pm \frac{1}{2}\left\{\left[\eta L - \frac{\tan\varphi_2(1+R)}{2\eta L}\right]^2 + 2R\tan\varphi_2\right\}^{1/2}$$

The amplitude ratio taken from equation (C18) has been rearranged to give ψ_G as follows:

$$\frac{\theta_{m2}}{\Delta T_G}\left\{\left[1 + \left(1 - \frac{x_G}{L}\right)\frac{\eta LR}{\psi_G}\right] + R\left(1 + \frac{x_G}{L}\frac{\eta L}{\psi_G}\right)\right\} - \left[1 + \left(1 - \frac{x_G}{L}\right)\frac{\eta LR}{\psi_G}\right]\cos\varphi_2 = 0$$

$$\frac{\theta_{m2}}{\Delta T_G}\left[\psi_G + \left(1 - \frac{x_G}{L}\right)\eta LR + R\psi_G + \frac{x_G}{L}\eta LR\right] - \left[\psi_G + \left(1 - \frac{x_G}{L}\right)\eta LR\right]\cos\varphi_2 = 0$$

$$\psi_G \left[\frac{\theta_{m2}}{\Delta T_G} (1 + R) - \cos \varphi_2 \right] + \frac{\theta_{m2}}{\Delta T_G} \left[\left(1 - \frac{x_G}{L}\right) \eta LR + \frac{x_G}{L} \eta LR \right] - \left(1 - \frac{x_G}{L}\right) \eta LR \cos \varphi_2 = 0$$

$$\psi_G = \frac{\eta LR \left\{ - \frac{\theta_{m2}}{\Delta T_G} \left[\left(1 - \frac{x_G}{L}\right) + \frac{x_G}{L} \right] + \left(1 - \frac{x_G}{L}\right) \cos \varphi_2 \right\}}{\frac{\theta_{m2}}{\Delta T_G} (1 + R) - \cos \varphi_2}$$

$$\psi_G = \frac{\eta LR \left[\left(1 - \frac{x_G}{L}\right) \cos \varphi_2 - \frac{\theta_{m2}}{\Delta T_G} \right]}{\frac{\theta_{m2}}{\Delta T_G} (1 + R) - \cos \varphi_2} \tag{C27}$$

This equation gives the convective heat-transfer coefficient ψ_G calculated using a lumped wall property system.

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